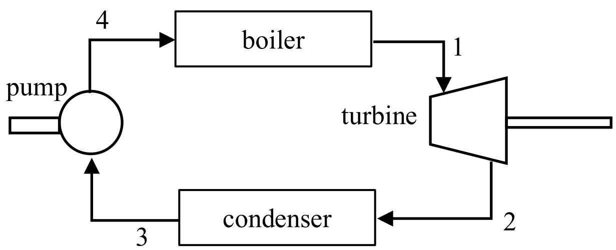
Consider a steam-power plant cycle in which saturated water vapor enters the turbine at 12.0 MPa (abs) and saturated liquid exits the condenser at a pressure of 0.012 MPa (abs). The net power output of the cycle is 122 MW.

a. Assuming that the isentropic efficiencies of the turbine and pump are 80%, determine the following:
   i. the mass flow rate of the water, in kg/h,
   ii. the rate of heat transfer into the boiler, in MW
   iii. the rate of heat transfer from the condenser, in MW, and
   iv. the thermal efficiency of the power plant cycle.
b. Draw a *T-s* diagram for the cycle, clearly indicating the process paths, states, and isobar values.



SOLUTION:
First determine the properties at each of the states.

At State 1:

We're given that the water is in a saturated vapor phase and p_1 = 12.0 MPa (abs) = 120 bar (abs). Using the Saturated Property Tables for water,

$T_1 = T_{1,sat}$ = 324.68 °C, $h_1 = h_{1g}$ = 2685.4 kJ/kg, and $s_1 = s_{1g}$ = 5.4939 kJ/(kg.K)

At State 2:

We're given that the turbine has an isentropic efficiency of 80%. In addition, since the pressure is assumed to remain constant across the condenser, $p_2 = p_3$ = 0.012 MPa (abs) = 0.12 bar (abs). At this pressure, interpolating from the Saturated Property Tables for water,

$T_2 = T_{2,sat}$ = 48.66 °C, h_{2f} = 203.73 kJ/kg, h_{2g} = 2588.9 kJ/kg, s_{2f} = 0.68576 kJ/(kg.K), and s_{2g} = 8.10048 kJ/(kg.K).

The isentropic efficiency of the turbine is given by,

$$\eta_{turbine,isen} \equiv \frac{\dot{W}_{by\ CV}}{\dot{W}_{by\ CV,isen}} = \frac{h_1 - h_2}{h_1 - h_{2s}}, \tag{1}$$

Thus,

$$h_2 = h_1 - \eta_{turbine,isen}(h_1 - h_{2s}), \tag{2}$$

To find h_{2s}, assume the turbine operates isentropically from 1 to 2, so that $s_{2s} = s_1$ = 5.4939 kJ/(kg.K). Thus,

$$x_{2s} = \frac{h_{2s} - h_{2f}}{h_{2g} - h_{2f}} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} \Rightarrow h_{2s} = h_{2f} + (h_{2g} - h_{2f})\left(\frac{s_{2s} - s_{2f}}{s_{2g} - s_{2f}}\right). \tag{3}$$

Using the values found previously, h_{2s} = 1748.33 kJ/kg. Substituting into Eq. (2) gives,

h_2 = 1937.41 kJ/kg.

The quality for this state is,

$$x_2 = \frac{h_2 - h_{2f}}{h_{2g} - h_{2f}} \Rightarrow x_2 = 0.7269. \tag{4}$$

The specific entropy at state 2 is then,

$$\Rightarrow s_2 = (1 - x_2)s_{2f} + x_2 s_{2g} \Rightarrow s_2 = 6.07521 \text{ kJ/(kg.k)}. \tag{5}$$

Note that $s_2 > s_1$, as expected for adiabatic operation of the turbine.

At State 3:

We're given that the water is in a saturated liquid phase and p_3 = 0.012 MPa (abs) = 0.12 bar (abs). Using the Saturated Property Tables for water and interpolating,

T_3 = 48.66 °C, h_3 = 203.73 kJ/kg, v_3 = 0.0010012 m^3/kg, and s_3 = 0.68576 kJ/(kg.K)

At State 4:

We're given that the pump has an isentropic efficiency of 80%. In addition, since the pressure is assumed to remain constant across the boiler, $p_1 = p_4$ = 12.0 MPa (abs) = 120 bar (abs).

The isentropic efficiency of the pump is given by,

$$\eta_{pump,isen} \equiv \frac{\dot{W}_{into\ CV,isen}}{\dot{W}_{into\ CV}} = \frac{h_{4s} - h_3}{h_4 - h_3}, \tag{6}$$

$$h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_{pump,isen}}, \tag{7}$$

Assuming an isentropic process from State 3 to State 4s (i.e., $s_{4s} = s_3$ = 0.68576 kJ/(kg.K)) and since the water can be treated as an incompressible substance at State 4s,

$$Tds = dh - vdp \Rightarrow dh = vdp \Rightarrow h_{4s} - h_3 = v_3(p_{4s} - p_3), \tag{8}$$

Using the parameters calculated previously, along with p_{4s} = 120 bar (abs),

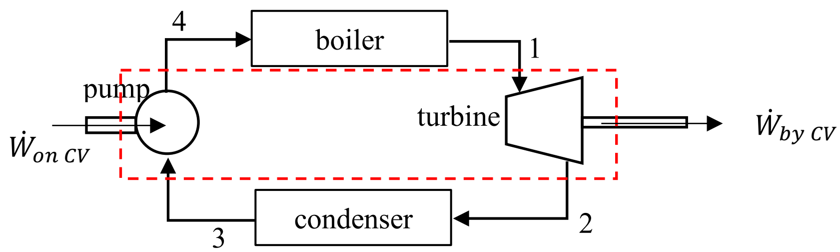
h_{4s} = 215.86 kJ/kg.

Using Eq. (7),
$h_4 =$ 218.89 kJ/kg.

Applying Conservation of Mass to individual control volumes surrounding each component and assuming steady flow gives,

$$\dot{m} = \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4. \quad (9)$$

Apply the 1st Law to a control volume surrounding the turbine and pump.



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{net,by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (10)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (11)$$

$$\dot{Q}_{into\ CV} = 0 \text{ (Assuming adiabatic operation.)}, \quad (12)$$

$$\dot{W}_{net,by\ CV} = \dot{W}_{by\ CV} - \dot{W}_{on\ CV}, \quad (13)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_3 - h_4 + h_1 - h_2). \quad (14)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

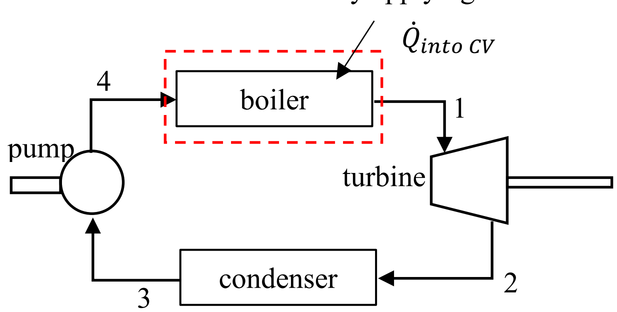
Substitute and simplify,

$$\dot{W}_{net,by\ CV} = \dot{m}(h_3 - h_4 + h_1 - h_2), \quad (15)$$

$$\boxed{\dot{m} = \frac{\dot{W}_{net,by\ CV}}{(h_3 - h_4 + h_1 - h_2)}}. \quad (16)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,
$\boxed{\dot{m} = 599 * 10^3 \text{ kg/h}}$.

The rate of heat transfer in the boiler is found by applying the 1st Law to a control volume surrounding the boiler.



$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \quad (17)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \quad (18)$$

$$\dot{W}_{by\ CV} = 0 \text{ (A boiler is a passive device.)}, \quad (19)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_4 - h_1). \quad (20)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

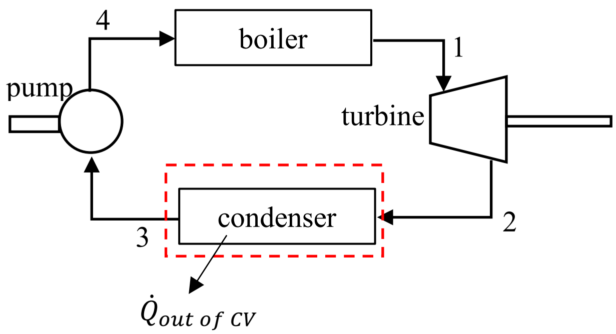
Substitute and simplify,

$$\boxed{\dot{Q}_{into\ CV} = \dot{m}(h_1 - h_4)}, \qquad (21)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,

$$\boxed{\dot{Q}_{into\ CV} = 411 \text{ MW}}.$$

To find the heat transfer from the condenser, apply the 1st Law to a control volume surrounding the condenser.



$$\frac{dE_{CV}}{dt} = -\dot{Q}_{out\ of\ CV} - \dot{W}_{by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \qquad (22)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \qquad (23)$$

$$\dot{W}_{by\ CV} = 0 \text{ (A condenser is a passive device.)}, \qquad (24)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_2 - h_3). \qquad (25)$$

(The changes in kinetic and potential energies are assumed to be negligibly small.)

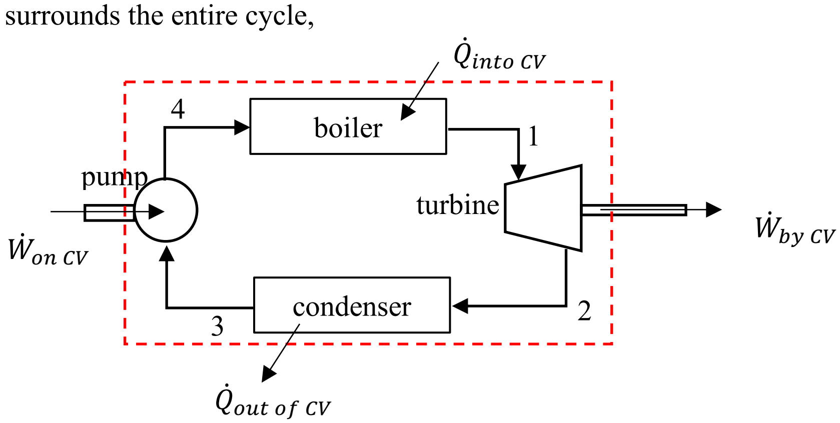
Substitute and simplify,

$$\boxed{\dot{Q}_{out\ of\ CV} = \dot{m}(h_2 - h_3)}, \qquad (26)$$

Using the parameters calculated previously in addition to the given net power output of 122 MW,

$$\boxed{\dot{Q}_{out\ of\ CV} = 289 \text{ MW}}.$$

Alternately, the rate of heat transfer out from the boiler could be found by applying the 1st Law to a control volume that surrounds the entire cycle,



$$\frac{dE_{CV}}{dt} = \dot{Q}_{net,into\ CV} - \dot{W}_{net,by\ CV} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \qquad (27)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (Assuming steady state operation.)}, \qquad (28)$$

$$\dot{Q}_{net,into\ CV} = \dot{Q}_{into\ CV} - \dot{Q}_{out\ of\ CV}, \qquad (29)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = 0, \tag{30}$$

(Since there is no mass transfer across the CV surface.)

Substitute and simplify,

$$\dot{W}_{net,by\ CV} = \dot{Q}_{into\ CV} - \dot{Q}_{out\ of\ CV}, \tag{31}$$

$$\boxed{\dot{Q}_{out\ of\ CV} = \dot{Q}_{into\ CV} - \dot{W}_{net,by\ CV}}. \tag{32}$$

Using the value found previously for the heat transfer into the control volume and given net power done by the cycle,

$$\boxed{\dot{Q}_{out\ of\ CV} = 289\text{ MW}},$$

which is the same value found previously.

The thermal efficiency of the power plant is,

$$\boxed{\eta \equiv \frac{\dot{W}_{net,by\ CV}}{\dot{Q}_{into\ CV}}}, \tag{33}$$

Using the parameters found previously,

$$\boxed{\eta = 0.297}.$$