Shown in the sketch below are various components of a refrigeration system operating on R-134a. The states are marked as 1, 2, 3, and 4 on the sketch. The refrigerant enters the evaporator (at state 4) as a liquid-vapor mixture where it absorbs heat from the refrigerated space. The pressure remains constant at 1.4 bar (abs) through the evaporator. The refrigerant then enters a well-insulated compressor at 1.4 bar (abs) with specific enthalpy of 243.40 kJ/kg (state 1). It leaves the compressor at 9 bar (abs) with specific enthalpy of 282.34 kJ/kg (state 2). The pressure remains constant at 9 bar (abs) through the condenser where heat is rejected to the environment. Saturated liquid exits the condenser at state 3, and then expands through a throttling valve (indicated on the figure as 'expansion valve') to the pressure of 1.4 bar (abs) at state 4. Assume steady-state operation. The mass flow rate of the refrigerant is 6 kg/min.

a. Draw a *T-v* diagram of the cycle indicating clearly the states 1, 2, 3, and 4.

Determine:
b. The compressor power (in kW).
c. The heat absorption rate in the evaporator (in kW).
d. The coefficient of performance of the refrigerator.
e. The quality of the mixture at state 4.
f. If the *cycle* satisfies the 1st Law. You must justify your conclusion.

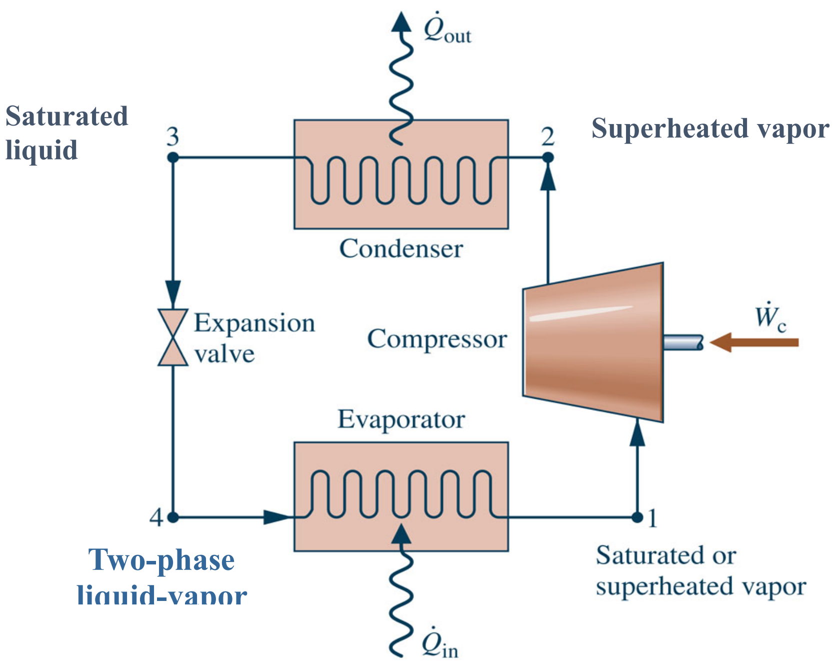


Table of Given Properties of R-134a at States 1-4

	1	2	3	4
Pressure [bar]	1.4	9	9	1.4
Enthalpy [kJ/kg]	243.40	282.34		
Phase		Superheated vapor	Saturated liquid	Liquid-vapor mixture

SOLUTION:
First, find the temperature and specific volumes at each of the states so that a *T*-*v* plot can be drawn.

At State 1:

p_1 = 1.4 bar and h_1 = 243.40 kJ/kg. Since $h_1 > h_{g@1.4\text{ bar}}$ = 239.18 kJ/kg (from the SLVM-pressure table), State 1 must be in a SHV phase. From the SHV table at p_1 = 1.4 bar and interpolating,

$$T_1 = \left(\frac{T_{-10\text{ °C}} - T_{-18.760\text{ °C}}}{h_{-10\text{ °C}} - h_{-18.760\text{ °C}}}\right)(h_1 - h_{-18.760\text{°C}}) + T_{-18.760\text{ °C}}, \tag{1}$$

where,

$T_{-18.760\text{ °C}}$ = -18.760 °C,
$T_{-10\text{ °C}}$ = -10 °C,
$h_{-18.760\text{ °C}}$ = 239.18 kJ/kg,
$h_{-10\text{ °C}}$ = 246.35 kJ/kg,
h_1 = 243.40 kJ/kg,
⇨ $\underline{T_1 = -13.6\text{ °C}}$.

Similarly, for the specific volume at State 1,

$$v_1 = \left(\frac{v_{-10\text{ °C}} - v_{-18.760\text{ °C}}}{h_{-10\text{ °C}} - h_{-18.760\text{ °C}}}\right)(h_1 - h_{-18.760\text{°C}}) + v_{-18.760\text{ °C}}, \tag{2}$$

where,

$v_{-18.760\text{ °C}}$ = 0.14015 m^3/kg,
$v_{-10\text{ °C}}$ = 0.14606 m^3/kg,
$h_{-18.760\text{ °C}}$ = 239.18 kJ/kg,
$h_{-10\text{ °C}}$ = 246.35 kJ/kg,
h_1 = 243.40 kJ/kg,
⇨ $\underline{v_1 = 0.1436\text{ m}^3\text{/kg}}$.

At State 2:

p_2 = 9 bar and h_2 = 282.34 kJ/kg. We're told that the R134a is a SHV so interpolating from the SHV table,

$$T_2 = \left(\frac{T_{50\text{ °C}} - T_{40\text{ °C}}}{h_{50\text{ °C}} - h_{40\text{ °C}}}\right)(h_2 - h_{40\text{°C}}) + T_{40\text{ °C}}, \tag{3}$$

where,

$T_{40\text{ °C}}$ = 40 °C,
$T_{50\text{ °C}}$ = 50 °C,
$h_{40\text{ °C}}$ = 274.17 kJ/kg,
$h_{50\text{ °C}}$ = 284.77 kJ/kg,
h_2 = 282.34 kJ/kg,
⇨ $\underline{T_2 = 47.7\text{ °C}}$.

Similarly, for the specific volume at State 1,

$$v_2 = \left(\frac{v_{50\text{ °C}} - v_{40\text{ °C}}}{h_{50\text{ °C}} - h_{40\text{ °C}}}\right)(h_2 - h_{40\text{°C}}) + v_{40\text{ °C}}, \tag{4}$$

where,

$v_{40\text{ °C}}$ = 0.023375 m^3/kg,
$v_{50\text{ °C}}$ = 0.024810 m^3/kg,
$h_{40\text{ °C}}$ = 274.17 kJ/kg,
$h_{50\text{ °C}}$ = 284.77 kJ/kg,
h_2 = 282.34 kJ/kg,
⇨ $\underline{v_2 = 0.02448\text{ m}^3\text{/kg}}$.

At State 3:

p_2 = 9 bar and a saturated liquid. From the SLVM-pressure table,

$\underline{T_3} = T_{sat} = \underline{35.526\text{ °C}}$,
$\underline{v_3} = v_f = \underline{0.00085811\text{ m}^3\text{/kg}}$,
$\underline{h_3} = h_f = \underline{101.64\text{ kJ/kg}}$.

At State 4:

p_4 = 1.4 bar and a SLVM. From the SLVM-pressure table,

$T_4 = T_{sat} = -18.760$ °C.

The other properties cannot yet be determined. To determine them, apply the 1st Law to a control volume surrounding the expansion valve,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{into} - \dot{W}_{by}, \quad (5)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (6)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_3 - h_4), \quad (7)$$

(Assuming steady flow, neglecting changes in KE and PE across the CV, and making use of COM to show that the mass flow rates at the inlet and outlet are identical.)

$$\dot{Q}_{into} = 0 \quad \text{(assuming flow through the valve occurs adiabatically)}, \quad (8)$$

$$\dot{W}_{by} = 0 \quad \text{(since the device is passive)}. \quad (9)$$

Substituting and simplifying gives,

$$h_4 = h_3. \quad (10)$$

Thus, $h_4 = 101.64$ kJ/kg. Use this value to determine the quality at State 4,

$$x_4 = \frac{h_4 - h_{4f}}{h_{4g} - h_{4f}}, \quad (11)$$

where,

$h_{4f} = 27.100$ kJ/kg and $h_{4g} = 239.18$ kJ/kg => $\boxed{x_4 = 0.351}$.

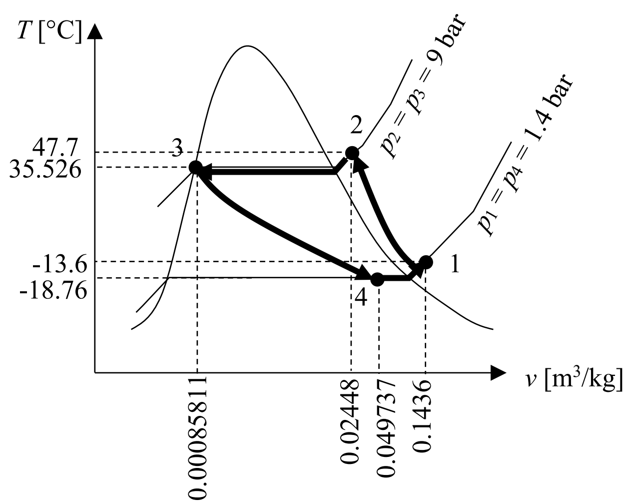
The specific volume at State 4 is,

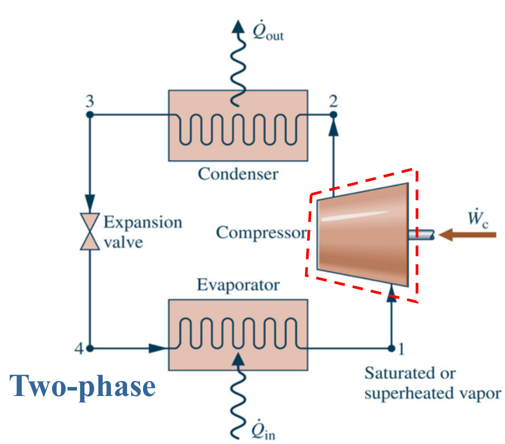
$$v_4 = (1 - x_4)v_{4f} + x_4 v_{4g}, \quad (12)$$

where,

$v_{4f} = 0.00073830$ m^3/kg and $v_{4g} = 0.14015$ m^3/kg => $v_4 = 0.049737$ m^3/kg.

Now that the temperatures and specific volumes at each of the states have been determined, plot the cycle on a *T*-*v* diagram.





The power required to operate the compressor is found by applying the 1st Law to a control volume surrounding the compressor,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{into} - \dot{W}_{by}, \quad (13)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (14)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_1 - h_2), \quad (15)$$

(Assuming steady flow, neglecting changes in KE and PE across the CV, and making use of COM to show that the mass flow rates at the inlet and outlet are identical.)

$$\dot{Q}_{into} = 0 \quad \text{(assuming flow through the compressor occurs adiabatically)}, \quad (16)$$

Substituting and simplifying gives,

$$\dot{W}_{by} = \dot{m}(h_1 - h_2). \quad (17)$$

Using the given and calculated values,

$\dot{m}$ = 6 kg/min = 0.1 kg/s, h_1 = 243.4 kJ/kg, and h_2 = 282.34 kJ/kg

=> $\boxed{\dot{W}_{by} = \text{-3.89 kW (3.89 kW of power goes into the compressor.)}}$

The heat absorption into the evaporator is found by applying the 1st Law to a control volume surrounding the evaporator,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{into} - \dot{W}_{by}, \quad (13)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (14)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_4 - h_1), \quad (15)$$

(Assuming steady flow, neglecting changes in KE and PE across the CV, and making use of COM to show that the mass flow rates at the inlet and outlet are identical.)

$$\dot{W}_{by} = 0 \quad \text{(the device is passive)}, \quad (16)$$

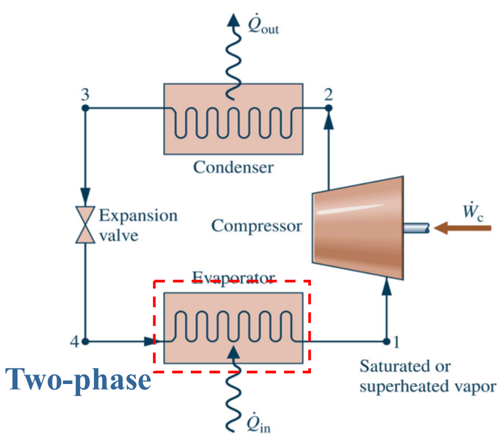
Substituting and simplifying gives,

$$\dot{Q}_{into} = \dot{m}(h_1 - h_4). \quad (17)$$

Using the given and calculated values,

$\dot{m}$ = 6 kg/min = 0.1 kg/s, h_1 = 243.4 kJ/kg, and h_4 = 101.64 kJ/kg

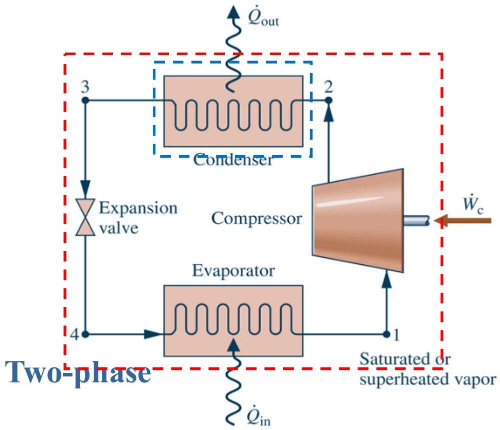
=> $\boxed{\dot{Q}_{into} = \text{14.2 kW (Heat goes into the evaporator.)}}$



The coefficient of performance for the refrigerator is,

$$COP_{ref} = \frac{\dot{Q}_{into\ evaporator}}{\dot{W}_{into}}, \quad (18)$$

Using the previously calculated values, $\boxed{COP_{ref} = 3.64}$.



We may calculate the rate of heat transfer from the condenser using a control volume surrounding the condenser (the blue control volume shown in the figure),

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) + \dot{Q}_{into} - \dot{W}_{by}, \quad (19)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (20)$$

$$\sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe) = \dot{m}(h_3 - h_2), \quad (21)$$

(Assuming steady flow, neglecting changes in KE and PE across the CV, and making use of COM to show that the mass flow rates at the inlet and outlet are identical.)

$$\dot{W}_{by} = 0 \quad \text{(the device is passive)}, \quad (22)$$

Substituting and simplifying gives,

$$\dot{Q}_{into} = \dot{m}(h_2 - h_3). \quad (23)$$

Using the given and calculated values,

$\dot{m}$ = 6 kg/min = 0.1 kg/s, h_3 = 101.64 kJ/kg, and h_2 = 282.34 kJ/kg

=> $\dot{Q}_{into}$ = -18.1 kW (Heat goes out of the condenser.)

Now apply the 1st Law to the entire cycle, i.e., apply the 1st Law to a control volume surrounding the entire cycle,

$$\left.\frac{dE_{CV}}{dt}\right|_{cycle} = \dot{Q}_{in,cycle} - \dot{W}_{by,cycle} \quad \text{(There's no mass flow into or out of the control volume.)}, \quad (24)$$

where,

$$\left.\frac{dE_{CV}}{dt}\right|_{cycle} = 0 \quad \text{(since, over a cycle, the initial state remains the same)}, \quad (25)$$

$$\dot{Q}_{in,cycle} = \dot{Q}_{in,evaporator} - \dot{Q}_{out,condenser}, \quad (26)$$

$$\dot{W}_{by,cycle} = \dot{W}_{by,compressor}. \quad (27)$$

Substitute and simplify,

$$0 = \dot{Q}_{in,evaporator} - \dot{Q}_{out,condenser} - \dot{W}_{by,compressor}, \quad (28)$$

Using the values calculated previously,

$\dot{Q}_{in,evaporator}$ = 14.2 kW,

$\dot{W}_{by,compressor}$ = -3.89 kW,

$\dot{Q}_{out,condenser}$ = 18.1 kW.

⇨ (14.2 kW) – (18.1 kW) – (-3.89 kW) = 0.

Thus, the 1st Law is satisfied using a CV surrounding the entire cycle.