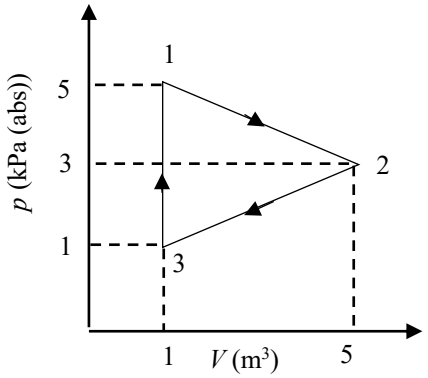
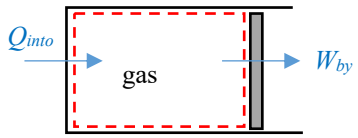
The figure below shows a power cycle (going from 1 to 2 to 3 and back to 1) executed by a gas in a piston-cylinder assembly. For process 1-2, $U_2 - U_1 = 15$ kJ and for process 3-1 $Q_{31} = 10$ kJ. There are no changes in kinetic or potential energy.



Determine:

a. the work for each process,
b. the heat transfer for processes 1-2 and 2-3, and
c. the thermal efficiency.

SOLUTION:



The (pdV) work done by the gas, i.e., the system, is,

$$W_{by\ sys} = \int_{V_{start}}^{V_{end}} pdV. \quad (1)$$

Note that the work done by the system will be the area under the p-V curve.

Since the process from 3 to 1 is at constant volume, there will be no work, i.e.,

$\boxed{W_{by\ sys,3\text{-}1} = 0}$.

The work from 1 to 2 is,

$$W_{by\ sys,1\text{-}2} = (3\text{ kPa})(5\text{ m}^3 - 1\text{ m}^3) + \tfrac{1}{2}*(5\text{ m}^3 - 1\text{ m}^3)*(5\text{ kPa} - 3\text{ kPa}), \quad (2)$$

$\boxed{W_{by\ sys,1\text{-}2} = 16\text{ kJ}}$.

The work from 2 to 3 is,

$$W_{by\ sys,1\text{-}2} = (1\text{ kPa})(1\text{ m}^3 - 5\text{ m}^3) + \tfrac{1}{2}*(1\text{ m}^3 - 5\text{ m}^3)*(3\text{ kPa} - 1\text{ kPa}), \quad (3)$$

$\boxed{W_{by\ sys,1\text{-}2} = -8\text{ kJ}}$.

The heat transfer into the system for process 1-2 may be found by applying the 1st Law,

$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}, \quad (4)$$

Since there are no changes in kinetic or potential energy, $\Delta E_{sys} = \Delta U_{sys}$. Thus,

$$Q_{into\ sys,1-2} = \Delta U_{sys,1-2} + W_{by\ sys,1-2}. \quad (5)$$

Using the given data,

$\Delta U_{1\text{-}2} = 15$ kJ,

⇨ $\boxed{Q_{into\ sys,1\text{-}2} = 31\text{ kJ}}$.

To find the heat transfer into the system for process 2-3, apply the 1st Law to the system over the entire cycle,

$$Q_{cycle,into\ sys} = W_{cycle,by\ sys}, \quad \text{(Recall that } \Delta E_{sys,cycle} = 0.) \quad (6)$$

where,

$$W_{cycle,by\ sys} = W_{1\text{-}2,by\ sys} + W_{2\text{-}3,by\ sys} + W_{3\text{-}1,by\ sys}, \quad (7)$$

$$Q_{cycle,into\ sys} = Q_{1\text{-}2,into\ sys} + Q_{2\text{-}3,into\ sys} + Q_{3\text{-}1,into\ sys}, \quad (8)$$

Using the previously calculated data, along with the given $Q_{3\text{-}1,into\ sys} = 10$ kJ,

⇨ $\boxed{Q_{2\text{-}3,into\ sys} = -33\text{ kJ}}$.

Note that $W_{cycle,by\ sys} = 8$ kJ $= W_{1\text{-}2,by\ sys} + W_{2\text{-}3,by\ sys} + W_{3\text{-}1,by\ sys}$ and $Q_{into\ sys} = Q_{1\text{-}2,into\ sys} + Q_{3\text{-}1,into\ sys} = 41$ kJ. This heat value is the heat transferred into the system. It is not over the entire cycle.

The thermal efficiency for this power cycle is,

$$\eta = \frac{W_{cycle,by\ sys}}{Q_{into\ sys}}, \quad (9)$$

Using the previously calculated values,

$\boxed{\eta = 19.5\%}$.