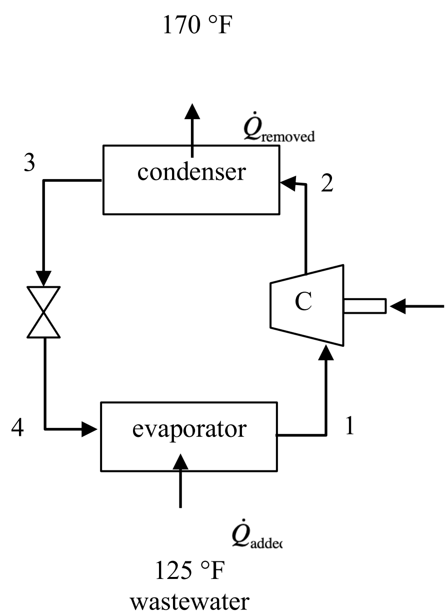
A process requires a heat transfer rate of $3*10^6$ Btu/h at 170 °F. It is proposed that a Refrigerant 134a vapor-compression heat pump be used to develop the process heating using a wastewater stream at 125 °F as the lower-temperature source. The compressor isentropic efficiency is 80%. Sketch the *T-s* diagram for the cycle and determine the:

a. specific enthalpy at the compressor exit, in Btu/lb_m,
b. temperatures at each of the principal states in °F,
c. mass flow rate of the refrigerant in lb_m/h,
d. compressor power, in Btu/h, and
e. coefficient of performance and compare with the coefficient of performance for a Carnot heat pump cycle operating between reservoirs at the process temperature and the wastewater temperature, respectively.



State	p [psia]	h [Btu/lb_m]
1	180	116.74
2	400	?
3	400	76.11
4	180	76.11

SOLUTION:

The specific enthalpy at state 2 may be found since the compressor efficiency is known ($\eta_{\text{comp}} = 0.80$),

$$\eta_{\text{comp}} \equiv \frac{\dot{W}_{\text{on comp},s}}{\dot{W}_{\text{on comp}}} = \frac{\dot{W}_{\text{on comp},s}/\dot{m}}{\dot{W}_{\text{on comp}}/\dot{m}} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_{\text{comp}}}, \tag{1}$$

where,

$s_{2s} = s_1 = 0.2154$ Btu/(lb$_m$.°R) using Table A-11E from Moran et al., 7th ed. with $p_1 = 180$ psia and knowing state 1 is in a saturated vapor state ($T_1 = 117.74$ °F)

$\Rightarrow h_{2s} = 123.32$ Btu/lb$_m$ knowing $p_{2s} = p_2 = 400$ psia (Table A-12E and interpolation; $T_{2s} = 186$ °F, SHV)

$\Rightarrow h_2 = 124.97$ Btu/lb$_m$ (using Table A-12E and interpolation $T_2 = 191.63$ °F, SHV)

Knowing $p_3 = p_2 = 400$ psia and $h_3 = 76.11$ Btu/lb$_m$ (saturated liquid state, Table A-11E) $\Rightarrow$ $T_3 = 179.95$ °F.
Knowing $p_4 = p_1 = 180$ psia and $h_4 = 76.11$ Btu/lb$_m$ (SLVM Table A-11E) $\Rightarrow$ $T_4 = 117.74$ °F.

Apply the 1st Law to the condenser to determine the mass flow rate,

$$\dot{Q}_{\text{removed}} = \dot{m}(h_2 - h_3) \Rightarrow \dot{m} = \frac{\dot{Q}_{\text{removed}}}{(h_2 - h_3)}, \tag{2}$$

$\Rightarrow$ $\dot{m} = 6.13 \times 10^4$ lb$_m$/h.

The compressor power is found by applying the 1st Law to the compressor,

$$\dot{W}_{\text{on comp}} = \dot{m}\left(h_2 - h_1\right), \quad (3)$$

$\Rightarrow$ $\boxed{\dot{W}_{\text{on comp}} = 5.05 \times 10^5 \text{ Btu/h}}$.

The coefficient of performance for the heat pump cycle is,

$$\text{COP}_{\text{hp}} \equiv \frac{\dot{Q}_{\text{removed}}}{\dot{W}_{\text{on comp}}}, \quad (4)$$

$\Rightarrow$ $\boxed{\text{COP}_{\text{hp}} = 5.95}$.

The COP_{hp} for the corresponding Carnot cycle operating between T_C = 125 °F (= 585 °R) and T_H = 170 °F (= 630 °R) is,

$$\text{COP}_{\text{hp,rev}} = \frac{T_H}{T_H - T_C}, \quad (5)$$

$\Rightarrow$ $\boxed{\text{COP}_{\text{hp,rev}} = 14}$.

The Carnot cycle COP is larger than the actual COP, as expected. Much of the cause for irreversibility in the actual system is due to the fact that the system temperatures from 2-3 and from 4-1 are substantially different than the hot and cold reservoir temperatures of T_H = 170 °F and T_C = 125 °F leading to large, finite temperature differences. Such large differences are needed for practical heat transfer rates between the condenser, evaporator, and the surroundings.

A sketch of the states and processes are shown on the following *T*-*s* diagram.