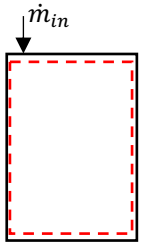
A homeowner connects a rain barrel to a downspout leading from the roof of their house. Assuming the downspout channels all of the water from 500 $ft^2$ of the roof area (the area when viewed from above), how long will it take to fill a 55 gallon rain barrel during a rain storm with a constant rain accumulation rate of 1 in. per hour?

SOLUTION:




Apply Conservation of Mass to a control volume surrounding the interior of the rain barrel.

$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$, (1)

where,

$\frac{dM_{CV}}{dt} = \frac{d(\rho V)}{dt} = \rho \frac{dV}{dt} = ?$, (2)

(where $\rho$ is the water density, which is assumed constant. The $V$ represents the volume of the water within the control volume),

$\sum_{in} \dot{m} = \rho \dot{h} A$, (3)

(where $\dot{h}$ is the accumulation rate on the roof and $A$ is the roof area)

$\sum_{out} \dot{m} = 0$ (The rain barrel outlet valve is assumed closed.) (4)

Substitute and solve for the rate of water volume increase in the rain barrel,

$\rho \frac{dV}{dt} = \rho \dot{h} A$ => $\frac{dV}{dt} = \dot{h} A$. (5)

Using the given values,

$\dot{h}$ = 1 in./hr = (1/12) ft/hr,

$A$ = 500 ft$^2$,

=> $dV/dt$ = 41.67 ft$^3$/hr = 311.69 gal/hr (1 ft$^3$ = 7.48052 gal).

Since the rain accumulation rate is a constant,

$V_T = (dV/dt)T$ => $T = V_T/(dV/dt)$, (6)

where $T$ is the time required to reach an accumulation volume of $V_T$. For $V_T$ = 55 gal, $T$ = 0.176 hr = 10.6 min.