

A can of compressed R134a, used as a “gas duster”, is discharged while in an upright orientation. Estimate the initial rate at which the specific internal energy changes within the can for the following conditions:

can volume = 295 ml,

R134a mass = 284 g,

initial can internal temperature = 20 °C,

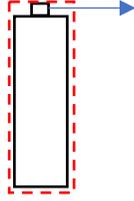
initial can internal pressure = 5.7171 bar (abs),

speed of the discharging jet = 11 m/s,

the cross-sectional area of the discharge nozzle = 1 mm<sup>2</sup>.



SOLUTION:



Apply the First Law to a control volume surrounding the contents within the can,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{into\ CV} - \dot{W}_{by\ CV,other} + \sum_{in} \dot{m}(h + ke + pe) - \sum_{out} \dot{m}(h + ke + pe), \quad (1)$$

where,

$$\frac{dE_{CV}}{dt} = \frac{dU_{CV}}{dt} = \frac{d}{dt}(m_{CV}u_{CV}) \quad (\text{where changes in } KE \text{ and } PE \text{ are negligible}), \quad (2)$$

$$\dot{Q}_{into\ CV} = 0 \quad (\text{assuming negligible heat transfer with the surroundings during discharge}), \quad (3)$$

$$\dot{W}_{by\ CV,other} = 0 \quad (\text{no work other than pressure work at the outlet}), \quad (4)$$

$$\sum_{in} \dot{m}(h + ke + pe) = 0 \quad (\text{there are no inlets into the can}), \quad (5)$$

$$\sum_{out} \dot{m}(h + ke + pe) = \dot{m}_{out}\left(h + \frac{1}{2}V^2\right)_{out}, \quad (6)$$

(there is one outlet; neglecting  $pe$  compared to  $h$  and  $ke$  at the outlet).

Substitute and simplify,

$$\frac{d}{dt}(m_{CV}u_{CV}) = -\dot{m}_{out}\left(h + \frac{1}{2}V^2\right)_{out}, \quad (7)$$

$$u_{CV} \frac{dm_{CV}}{dt} + m_{CV} \frac{du_{CV}}{dt} = -\dot{m}_{out}\left(h + \frac{1}{2}V^2\right)_{out}. \quad (8)$$

Now apply Conservation of Mass to the same control volume,

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \quad (9)$$

where,

$$\sum_{in} \dot{m} = 0 \quad (\text{there are no inlets into the can}), \quad (10)$$

$$\sum_{out} \dot{m} = \dot{m}_{out}, \quad (11)$$

$$\Rightarrow \frac{dM_{CV}}{dt} = -\dot{m}_{out}. \quad (12)$$

Substitute Eq. (12) into Eq. (8),

$$-u_{CV}\dot{m}_{out} + m_{CV} \frac{du_{CV}}{dt} = -\dot{m}_{out}\left(h + \frac{1}{2}V^2\right)_{out}, \quad (13)$$

$$\frac{du_{CV}}{dt} = \frac{-\dot{m}_{out}\left(h_{out} + \frac{1}{2}V_{out}^2 - u_{CV}\right)}{m_{CV}}. \quad (14)$$

Note that the mass flow rate out of the control volume is,

$$\dot{m}_{out} = \frac{V_{out}A_{out}}{v_{out}}, \quad (15)$$

where  $A_{out}$  and  $v_{out}$  are the cross-sectional area of the nozzle outlet and the specific volume of the R134a at the outlet, respectively.

Now determine the properties at the initial state,

$$\begin{aligned} m_{CV,i} &= 284 \text{ g} = 0.284 \text{ kg}, \\ V_i &= 295 \text{ ml} = 2.95 \cdot 10^{-4} \text{ m}^3, \\ p_i &= 5.7171 \text{ bar (abs)}, \\ T_i &= 20 \text{ }^\circ\text{C}. \end{aligned}$$

Temp. (C)	Press. (bar)	Liquid				Vapor			
		Volume ( $v_f$ , m <sup>3</sup> /kg)	Internal Energy ( $u_f$ , kJ/kg)	Enthalpy ( $h_f$ , kJ/kg)	Entropy ( $s_f$ , kJ/kg/K)	Volume ( $v_g$ , m <sup>3</sup> /kg)	Internal Energy ( $u_g$ , kJ/kg)	Enthalpy ( $h_g$ , kJ/kg)	Entropy ( $s_g$ , kJ/kg/K)
20	5.7171	0.00081610	78.857	79.324	0.30063	0.035997	241.02	261.60	0.92243

From the property tables for R134a at this pressure and temperature, we observe that the R134a is in a SLVM phase. The specific volume is,

$$\Rightarrow v_{CV,i} = \frac{V_i}{m_i} = 1.04 \cdot 10^{-3} \text{ m}^3/\text{kg}. \quad (16)$$

The initial quality is,

$$x_i = \frac{v_i - v_{l,i}}{v_{v,i} - v_{l,i}} \quad (17)$$

where,

$$\begin{aligned} v_{l,i} &= 0.00081610 \text{ m}^3/\text{kg}, \\ v_{v,i} &= 0.035997 \text{ m}^3/\text{kg}, \\ \Rightarrow x_i &= 6.33 \cdot 10^{-3}. \text{ (The can is initially mostly liquid by mass.)} \end{aligned}$$

The corresponding specific internal energy within the control volume is,

$$u_{CV,i} = (1 - x_i)u_{l,i} + x_i u_{v,i}, \quad (18)$$

where,

$$\begin{aligned} u_{l,i} &= 78.857 \text{ kJ/kg}, \\ u_{v,i} &= 241.02 \text{ kJ/kg}, \\ \Rightarrow u_{CV,i} &= 79.88 \text{ kJ/kg}. \end{aligned}$$

Since the can is upright when discharged, it will discharge vapor (vapor at the top of the can, liquid at the bottom).

Thus,

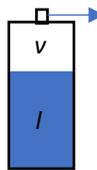
$$\begin{aligned} h_{out,i} &= h_{v,i} = 261.60 \text{ kJ/kg}, \\ v_{out,i} &= v_{v,i} = 0.035997 \text{ m}^3/\text{kg}. \end{aligned}$$

We're also given that,

$$\begin{aligned} V_{out} &= 11 \text{ m}^3/\text{s} \quad (\Rightarrow \frac{1}{2} V_{out}^2 = 60.5 \text{ m}^2/\text{s}^2) \\ A_{out} &= 1 \text{ mm}^2 = 1 \cdot 10^{-6} \text{ m}^2. \end{aligned}$$

Thus, from Eq. (15),

$$\dot{m}_{out} = 3.0558 \cdot 10^{-4} \text{ kg/s}.$$



Using all the previously calculated values in Eq. (14),

$$\frac{du_{CV,i}}{dt} = -8.72 \cdot 10^{-2} \text{ kJ/kg/s}.$$

Thus, the internal energy in the can is decreasing with time.

Without an analytical relationship between the properties for R134a (they're only available to us in introductory thermodynamics in tabular form), we cannot easily determine the rate of temperature or pressure change in the can. Instead, we'd need to approximate the rates of change numerically for calculating derivatives. Using a simple Euler scheme, for example, the rate at which the specific volume changes is,

$$\frac{dv_{CV,t=0}}{dt} \approx \frac{v_{CV,t=\Delta t} - v_{CV,t=0}}{\Delta t - 0} \Rightarrow v_{CV,t=\Delta t} = v_{CV,t=0} + \frac{dv_{CV,t=0}}{dt} \Delta t, \quad (19)$$

where  $\Delta t$  needs to be a small value for numerical accuracy (and stability). In this equation, the rate at which the specific volume changes is,

$$\frac{dv_{CV}}{dt} = \frac{d}{dt} \left( \frac{V_{CV}}{m_{CV}} \right) = - \frac{V_{CV}}{m_{CV}^2} \frac{dm_{CV}}{dt} \quad (\text{since the volume is constant, but the mass changes}). \quad (20)$$

Similarly, for the specific internal energy,

$$\frac{du_{CV,t=0}}{dt} \approx \frac{u_{CV,t=\Delta t} - u_{CV,t=0}}{\Delta t - 0} \Rightarrow u_{CV,t=\Delta t} = u_{CV,t=0} + \frac{du_{CV,t=0}}{dt} \Delta t. \quad (21)$$

Using the previously determined values and a small value of  $\Delta t$ , one could calculate the specific volume and specific internal energy at the next time ( $t = 0 + \Delta t$ ). From these properties, one could then determine the remainder of the properties, e.g., temperature and pressure, from the property table. This approach is obviously tedious to perform by hand. Normally, one would instead fit an equation to the tabular data and then use that equation to estimate how the properties change, e.g.,

$$T = f(v, u) \Rightarrow \frac{dT}{dt} = \frac{\partial f}{\partial v} \frac{dv}{dt} + \frac{\partial f}{\partial u} \frac{du}{dt}. \quad (22)$$