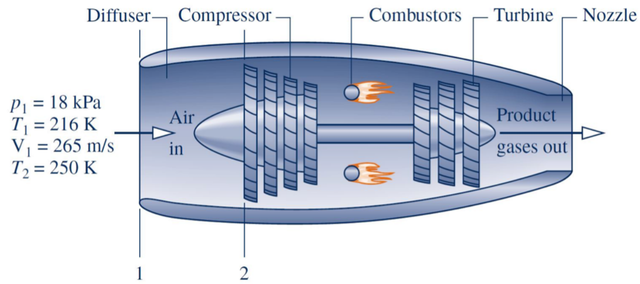
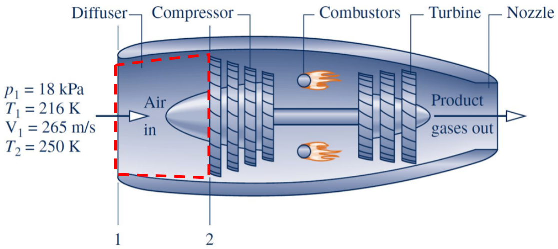
As shown in the following figure, air enters the diffuser of a jet engine operating at steady state at 18 kPa (abs), 216 K, and a velocity of 265 m/s, all data corresponding to high-altitude flight. The air flows adiabatically through the diffuser and achieves a temperature of 250 K at the diffuser exit. Using the ideal gas model for air, determine the velocity of the air at the diffuser exit, in m/s.



SOLUTION:



Apply Conservation of Mass to the control volume shown in the figure,

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \qquad (1)$$

where,

$$\frac{dM_{CV}}{dt} = 0 \quad \text{(assume steady state operation)}, \qquad (2)$$

$$\sum_{in} \dot{m} - \sum_{out} \dot{m} = \dot{m}_1 - \dot{m}_2. \qquad (3)$$

Substituting and simplifying,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}. \qquad (4)$$

Now apply the First Law to the same control volume,

$$\frac{dE_{CV}}{dt} = \dot{Q}_{into} - \dot{W}_{by} + \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right), \qquad (5)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assume steady state operation)}, \qquad (6)$$

$$\dot{Q}_{into} = 0 \quad \text{(assume an adiabatic diffuser)}, \qquad (7)$$

$$\dot{W}_{by} = 0 \quad \text{(a diffuser is a passive device)}, \qquad (8)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}\left[\left(h + \tfrac{1}{2}V^2\right)_1 - \left(h + \tfrac{1}{2}V^2\right)_2\right] \qquad (9)$$

(neglecting the *pe* contribution since we're dealing with a gas over small elevation difference).

Substitute and simplify,

$$0 = \dot{m}\left[\left(h + \tfrac{1}{2}V^2\right)_1 - \left(h + \tfrac{1}{2}V^2\right)_2\right], \qquad (10)$$

$$\tfrac{1}{2}V_2^2 = \tfrac{1}{2}V_1^2 + (h_1 - h_2). \qquad (11)$$

Using the given parameters,

V_1 = 265 m/s,

T_1 = 216 K => h_1 = 216.0 kJ/kg (using the Ideal Gas Table for air), (12)

T_2 = 250 K => h_2 = 250.0 kJ/kg (using the Ideal Gas Table for air), (13)

=> $\boxed{V_2 = 47.2 \text{ m/s}}$. (Note that 1000 m^2/s^2 = 1 kJ/kg.)