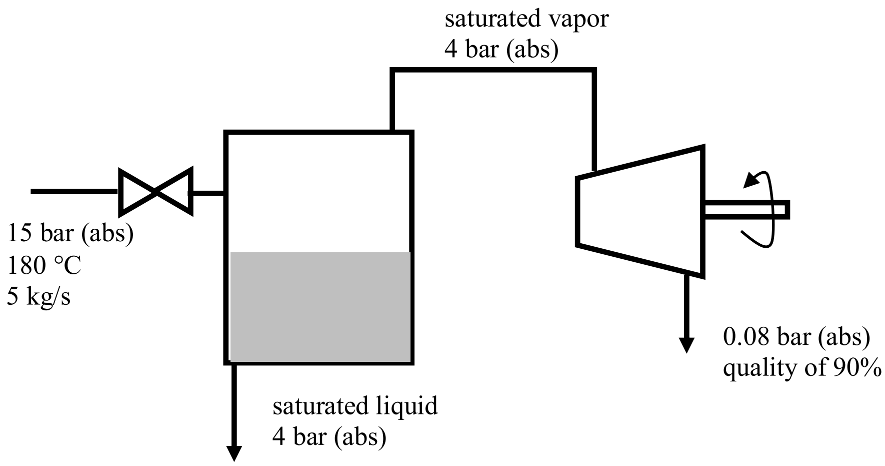
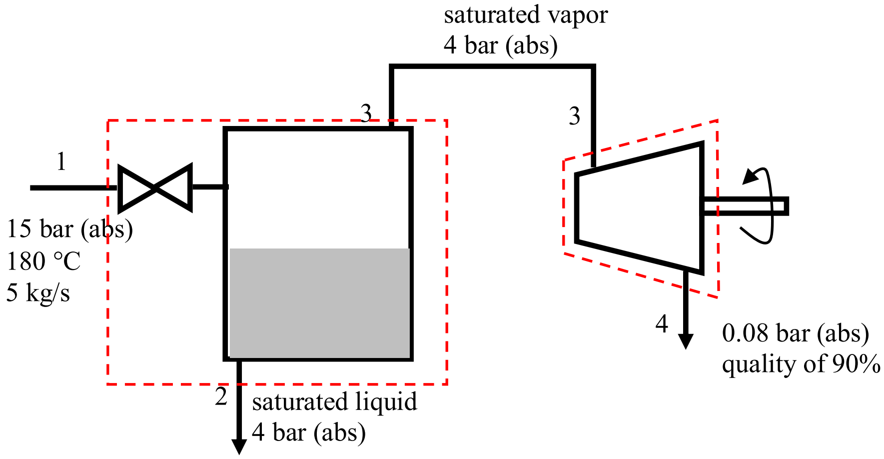
Hot industrial waste water at 15 bar (abs), 180 °C with a mass flow rate of 5 kg/s enters a flash chamber via a valve. Saturated vapor and saturated liquid streams, each at 4 bar (abs), exit the flash chamber. The saturated vapor enters the turbine and expands to 0.08 bar (abs) with a quality of 90%. Stray heat transfer and kinetic and potential energy effects are negligible. For operation at steady state, determine the power developed by the turbine.



SOLUTION:



Apply the 1st Law to a control volume surrounding the turbine,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added} - \dot{W}_{by,other}, \quad (1)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (2)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_3(h_3 - h_4) \quad \text{(neglecting changes in KE and PE)}, \quad (3)$$

$$\dot{Q}_{added} = 0 \quad \text{(assuming the turbine is well insulated)}, \quad (4)$$

$$\dot{W}_{by,other} = ? \quad (5)$$

Substitute and simplify,

$$0 = \dot{m}_3(h_3 - h_4) - \dot{W}_{by,34}, \quad (6)$$

$$\dot{W}_{by,34} = \dot{m}_3(h_3 - h_4). \quad (7)$$

Determine the specific enthalpies using the SLVM table for water,

State 3: saturated vapor at 4 bar (abs) => $h_3 = 2738.1$ kJ/kg,

State 4: $h_4 = (1 - x_4)h_{f4} + x_4 h_{g4}$ (8)

where $x_4 = 0.90$ (given) and $h_{f4} = 173.84$ kJ/kg and $h_{g4} = 2576.2$ kJ/kg at $p_{sat} = 0.08$ bar (abs),

⇨ $h_4 = 2336$ kJ/kg.

The mass flow rate at State 3 may be found by applying a combination of Conservation of Mass and the 1st Law to a control volume surrounding the flash chamber and valve,

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \quad (9)$$

where,

$$\frac{dM_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (10)$$

$$\sum_{in} \dot{m} - \sum_{out} \dot{m} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3. \quad (11)$$

Substitute and simplify,

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3. \quad (12)$$

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added} - \dot{W}_{by,other}, \quad (13)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (14)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3, \quad (15)$$

(neglecting changes in KE and PE),

$$\dot{Q}_{added} = 0 \quad \text{(assuming the flash chamber is well insulated)}, \quad (16)$$

$$\dot{W}_{by,other} = 0 \quad \text{(the flash chamber is a passive device)}, \quad (17)$$

Substitute and simplify,

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3, \quad (18)$$

$$\dot{m}_3 h_3 = \dot{m}_1 h_1 - \dot{m}_2 h_2, \quad (19)$$

$$\dot{m}_3 h_3 = \dot{m}_1 h_1 - (\dot{m}_1 - \dot{m}_3) h_2, \quad \text{(Making use of Eq. (12).)} \quad (20)$$

$$\dot{m}_3 = \dot{m}_1 \left(\frac{h_1 - h_2}{h_3 - h_2}\right), \quad \text{(Making use of Eq. (12).)} \quad (21)$$

Determine the specific enthalpies at States 1 and 2 using the property tables for water,

$h_2 = 604.65$ kJ/kg (saturated liquid at $p_{sat} = 4$ bar (abs)),

$h_1 \approx h_f(T_1) + v_f(T_1)[p_1 - p_{sat}(T_1)]$, (a compressed liquid since $T_1 < T_{sat,\,@\,15\text{ bar}}$),

where $h_f(T_1) = 763.05$ kJ/kg, $v_f(T_1) = 0.0011274$ m^3/kg, $p_1 = 15$ bar (abs), $p_{sat}(T_1) = 10.028$ bar (abs),

⇨ $h_1 = 763.1$ kJ/kg.

Using the given values, Eq. (21) gives,

$\dot{m}_3 = 0.371$ kg/s.

Using this value and previously calculated values, Eq. (7) gives,

$\dot{W}_{by,34} = 149$ kW. (The turbine generates 149 kW of power.)