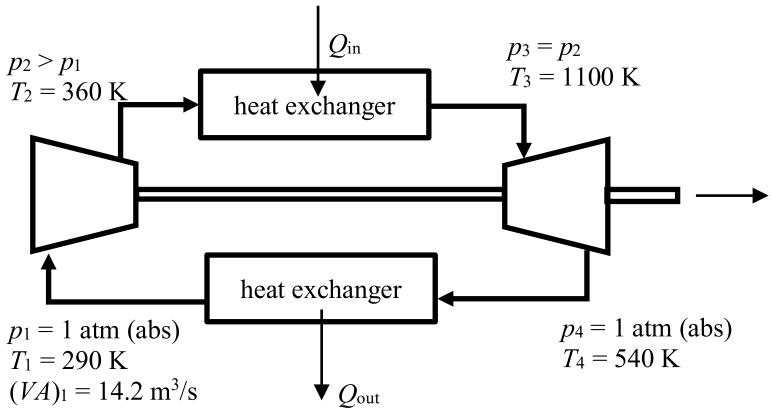
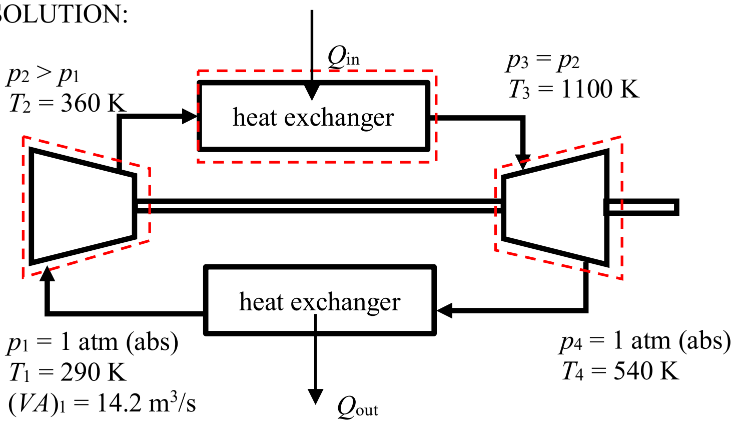
A simple gas turbine power cycle operating at steady state with air as the working substance is shown in the figure. The cycle components include an air compressor mounted on the same shaft as the turbine. The air is heated in the high pressure heat exchanger before entering the turbine. The air exiting the turbine is cooled in the low-pressure heat exchanger before returning to the compressor. Kinetic and potential energy effects are negligible. The compressor and turbine are adiabatic. Using the ideal gas model for air, determine:

a. the power required for the compressor,
b. power output of the turbine, and
c. the thermal efficiency of the cycle.



SOLUTION:



From Conservation of Mass applied to control volumes surrounding each component, assuming steady state,

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 = \dot{m}, \quad (1)$$

The mass flow rate may be found from the data given at State 1,

$$\dot{m} = \frac{(VA)_1}{v_1}, \quad (2)$$

where, assuming that the air behaves as an ideal gas,

$$v_1 = \frac{R_{air}T_1}{p_1}. \quad (3)$$

Using the given data,

$R_{air} = 287$ J/(kg.K),
$T_1 = 290$ K,
$p_1 = 1$ atm (abs) $= 101 \times 10^3$ Pa,
⇨ $v_1 = 0.824$ m³/kg,
⇨ $\dot{m} = 17.2$ kg/s.

Now apply the 1st Law to a control volume surrounding the compressor,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added} - \dot{W}_{by,other}, \quad (4)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (5)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_1 - h_2) \quad \text{(neglected changes in KE and PE)}, \quad (6)$$

$$\dot{Q}_{added} = 0 \quad \text{(assuming the compressor is well insulated)}, \quad (7)$$

$$\dot{W}_{by,other} = ? \quad (8)$$

Substitute and simplify,

$$0 = \dot{m}(h_1 - h_2) - \dot{W}_{by,other}, \quad (9)$$

$$\dot{W}_{by,other} = \dot{m}(h_1 - h_2). \quad (10)$$

Use the ideal gas table for air to determine the specific enthalpies at the given temperatures,

$h_1 = h(T_1 = 290 \text{ K}) = 290.1$ kJ/kg,
$h_2 = h(T_2 = 360 \text{ K}) = 360.6$ kJ/kg,
⇨ $\boxed{\dot{W}_{by,other} = \text{-1210 kW. (The compressor requires 1210 kW.)}}$

To find the power output of the turbine, apply the 1st Law to a control volume surrounding the turbine,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added} - \dot{W}_{by,other}, \quad (11)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (12)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_3 - h_4) \quad \text{(neglected changes in KE and PE)}, \quad (13)$$

$$\dot{Q}_{added} = 0 \quad \text{(assuming the turbine is well insulated)}, \quad (14)$$

$$\dot{W}_{by,other} = ? \quad (15)$$

Substitute and simplify,

$$0 = \dot{m}(h_3 - h_4) - \dot{W}_{by,other}, \quad (16)$$

$$\dot{W}_{by,other} = \dot{m}(h_3 - h_4). \quad (17)$$

Use the ideal gas table for air to determine the specific enthalpies at the given temperatures,

$h_3 = h(T_3 = 1100 \text{ K}) = 1161$ kJ/kg,

$h_4 = h(T_4 = 540 \text{ K}) = 544.6$ kJ/kg,

⇨ $\boxed{\dot{W}_{by,other} = 10600 \text{ kW. (The turbine generates 10600 kW of power.)}}$

In order to determine the thermal efficiency of the cycle, we must also determine the heat added to the cycle. The added heat may be found by applying the 1st Law to a control volume surrounding the top heat exchanger,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added} - \dot{W}_{by,other}, \quad (18)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(assuming steady state)}, \quad (19)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}(h_2 - h_3) \quad \text{(neglected changes in KE and PE)}, \quad (20)$$

$$\dot{Q}_{added} = ?, \quad (21)$$

$$\dot{W}_{by,other} = 0 \quad \text{(The heat exchanger is a passive device.)} \quad (22)$$

Substitute and simplify,

$$0 = \dot{m}(h_2 - h_3) + \dot{Q}_{added}, \quad (23)$$

$$\dot{Q}_{added} = \dot{m}(h_3 - h_2). \quad (24)$$

Using the previously found specific enthalpies,

$\dot{Q}_{added} = 13800$ kW. (13800 kW of heat is added to the heat exchanger.)

The thermal efficiency of the cycle is given by,

$$\eta = \frac{\dot{W}_{net,by}}{\dot{Q}_{net,added}} = \frac{\dot{W}_{by,12} + \dot{W}_{by,34}}{\dot{Q}_{added,23}}, \quad (25)$$

Using the previously calculated values,

⇨ $\boxed{\eta = 0.682}$.