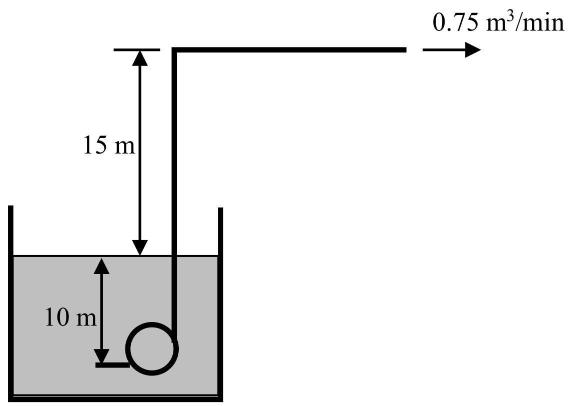
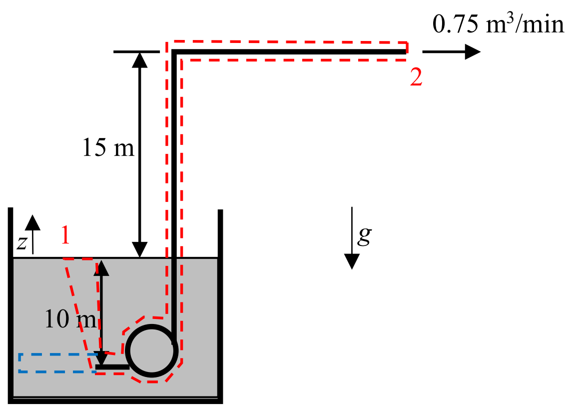
Steady-state operating data are shown for the submerged pump and delivery pipe system shown in the figure. At the inlet, the volumetric flow rate is 0.75 m^3/min and the temperature is 15 °C. At the exit, the pressure is 1 atm (abs). There is no significant change in water temperature or kinetic energy from inlet to exit. Heat transfer between the pump and its surrounding is negligible. Determine the minimum power required by the pump.



SOLUTION:

Apply Conservation of Mass to a control volume (the red one) that encompasses part of the free surface, pump, and pipe, as shown in the figure below. Note that the control volume between the pump inlet and the free surface of the tank is parallel to flow streamlines such that there's no flow across the control volume boundary. A control volume that crosses the inlet of the pipe was not used since the pressure at that location is not well known. Since the fluid velocity is not zero there, the pressure is not necessarily equal to the hydrostatic pressure at that location. A control volume that extends far in front of the inlet pipe (the blue one) would be acceptable since the velocity far from the pipe inlet would be nearly zero and the pressure would then be hydrostatic.



$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \tag{1}$$

where,

$$\frac{dM_{CV}}{dt} = 0, \quad \text{(Assuming steady state.)} \tag{2}$$

so that,

$$\dot{m}_2 = \dot{m}_1 = \dot{m}. \tag{3}$$

Now apply the 1st Law to the same control volume,

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{into\ CV} - \dot{W}_{\substack{by\ CV,\\ other}}, \tag{1}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(Assuming steady state.)}, \tag{2}$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}\left[(h_1 - h_2) - \tfrac{1}{2}(V_1^2 - V_2^2) + g(z_1 - z_2)\right], \tag{3}$$

$$\dot{Q}_{into\ CV} = 0 \quad \text{(We're told to neglect heat transfer.)}, \tag{4}$$

$$\dot{W}_{\substack{by\ CV,\\ other}} = ? \quad \text{(Trying to find this quantity.)} \tag{5}$$

Also note that because we're dealing with a compressed liquid,

$$h_{CL}(T, p) \approx h_f(T) + v_f(T)[p - p_{sat}(T)]. \tag{6}$$

Since we're told to assume the temperature change of the water is negligible,

$$h_1 - h_2 \approx v_f(T)(p_1 - p_2). \tag{7}$$

Note that since states 1 and 2 are exposed to the atmosphere, $p_1 = p_2 = p_{atm} \Rightarrow h_1 - h_2 = 0$.

We're also told to neglect the kinetic energy differences between states 1 and 2 => $\frac{1}{2}(V_1^2 - V_2^2) \approx 0$.

Using these additional simplifications, the 1st Law simplifies to,

$$\dot{W}_{\underset{other}{by\ CV,}} = \dot{m}g(z_1 - z_2). \tag{8}$$

For the given data,

Q = 0.75 m^3/min = 0.0125 m^3/s,

T = 15 °C => $v_f(T)$ = 0.0010009 m^3/kg (Note: $v_{CL}(T, p) \approx v_f(T)$.)

$\dot{m} = \frac{Q}{v}$ => $\dot{m} = 12.49$ kg/s,

g = 9.81 m/s^2,

z_1 = 0,

z_2 = 15 m,

⇨ $\dot{W}_{\underset{other}{by\ CV,}} = -1840\ W,$

i.e., $\boxed{\text{1.84 kW must be added to the control volume (pump) for the given flow scenario}}$.

Note that if the blue control volume is used, then

z_1 = -10 m and

$p_1 = p_{atm} + gz_1/v$ and $p_2 = p_{atm}$ (discharging to atmosphere)

Substituting these values into the 1st Law gives the same answer found previously.