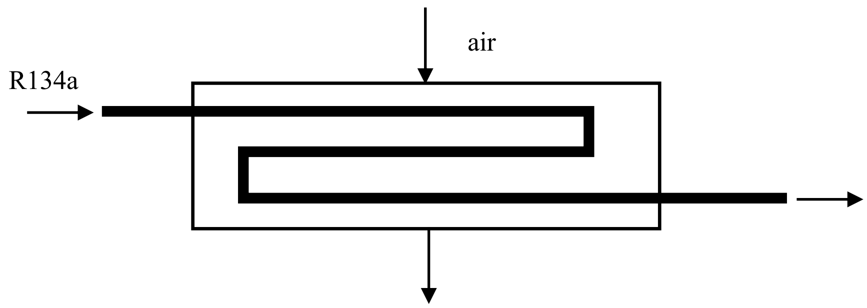
An air-conditioning system is shown in which air flows over tubes carrying R134a. Air enters with a volumetric flow rate of 50 m^3/min at 32 °C and 1 bar (abs) and exits at 22 °C and 0.95 bar (abs). R134a enters the tubes at 5 bar (abs) with a quality of 20% and exits at 5 bar (abs) and 20 °C. Ignoring heat transfer at the outer surface of the air conditioner and neglecting kinetic and potential energy effects, determine at steady state,

a. the mass flow rate of the R134a, and
b. the rate of heat transfer between the air and refrigerant.



SOLUTION:
First, apply conservation of mass to a control volume that surrounds only the R134a pipe (diagram not shown).

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \quad (1)$$

where,

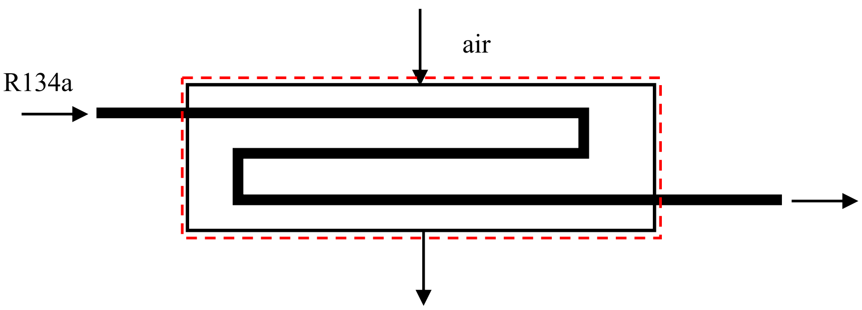
$$\frac{dM_{CV,R134a}}{dt} = 0 \text{ (steady state)}, \quad (2)$$

$$\Rightarrow \dot{m}_{out,R134a} = \dot{m}_{in,R134a} = \dot{m}_{R134a}. \quad (3)$$

Similarly, applying conservation of mass to a control volume consisting of just the air,

$$\dot{m}_{out,air} = \dot{m}_{in,air} = \dot{m}_{air}. \quad (4)$$

Now apply the 1st Law to a control volume surrounding the entire air-conditioning system.



$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added\ to\ CV} - \dot{W}_{by\ CV,other}, \quad (5)$$

where,

$$\frac{dE_{CV}}{dt} = 0 \text{ (steady state)}, \quad (6)$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_{R134a} h_{R134a,in} + \dot{m}_{air} h_{air,in}, \quad (7)$$

$$\sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_{R134a} h_{R134a,out} + \dot{m}_{air} h_{air,out}, \quad (8)$$

Note that changes in kinetic and potential energies are assumed negligible.

$$\dot{Q}_{added\ to\ CV} = 0 \text{ (well insulated, i.e., adiabatic)}, \quad (9)$$

$$\dot{W}_{by\ CV,other} = 0 \text{ (no work other than pressure work at the inlets and outlets)}. \quad (10)$$

Substitute and simplify,

$$\left(\dot{m}_{R134a} h_{R134a,in} + \dot{m}_{air} h_{air,in}\right) - \left(\dot{m}_{R134a} h_{R134a,out} + \dot{m}_{air} h_{air,out}\right) = 0, \quad (11)$$

$$\dot{m}_{R134a}\left(h_{R134a,in} - h_{R134a,out}\right) - \dot{m}_{air}\left(h_{air,out} - h_{air,in}\right) = 0, \quad (12)$$

$$\dot{m}_{R134a} = \dot{m}_{air}\left(\frac{h_{air,out} - h_{air,in}}{h_{R134a,in} - h_{R134a,out}}\right). \quad (13)$$

The specific enthalpies for the air may be found by assuming the air is an ideal gas and using the ideal gas tables,

$h_{air,out}$ = 295.1 kJ/kg (ideal gas table for air at $T_{air,out}$ = 22 °C = 295 K),

$h_{air,in}$ = 305.2 kJ/kg (ideal gas table for air at $T_{air,in}$ = 32 °C = 305 K).

The specific enthalpies for the R134a are found using the properties tables,

$h_{R134a,in} = (1 - x_{R134a,in})h_{R134,in,f} + x_{R134a,in}h_{R134,in,g}$ = 110.55 kJ/kg

where $x_{R134a,in}$ = 0.20 and, at $p_{sat,in}$ = 5 bar (abs) (T_{sat} = 15.735 °C), $h_{R134a,in,f}$ = 73.358 kJ/kg, $h_{R134a,in,g}$ = 259.33 kJ/kg

$h_{R134a,out}$ = 263.5 kJ/kg since at p_{out} = 5 bar (abs), T_{sat} = 15.735 °C < T_{out} = 20 °C => the R134a at the outlet is a superheated vapor.

The air mass flow rate may be found from the given air volumetric flow rate at the inlet,

$$\dot{m}_{air} = \frac{Q_{air,in}}{v_{air,in}}, \tag{14}$$

where,

$Q_{air,in}$ = 50 m^3/min,

$v_{air,in} = R_{air}T_{air,in}/p_{air,in}$ = [287 J/(kg.K)].[305 K]/[100*10^3 Pa] = 0.8754 m^3/kg,

⇨ $\dot{m}_{air}$ = 57.12 kg/min

Substituting the specific enthalpy values along with the mass flow rate for the air,

⇨ $\boxed{\dot{m}_{R134a} = 3.77 \text{ kg/min}}$

To find the rate of heat transfer between the air and refrigerant, apply the 1st Law to a control volume surrounding just the air (not shown here).

$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{added\ to\ CV} - \dot{W}_{by\ CV,other}, \tag{15}$$

where,

$$\frac{dE_{CV}}{dt} = 0 \quad \text{(steady state)}, \tag{16}$$

$$\sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_{air}h_{air,in}, \tag{17}$$

$$\sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) = \dot{m}_{air}h_{air,out}, \tag{18}$$

Note that changes in kinetic and potential energies are assumed negligible.

$$\dot{Q}_{added\ to\ CV} \quad \text{(trying to find this quantity)}, \tag{19}$$

$$\dot{W}_{by\ CV,other} = 0 \quad \text{(no work other than pressure work at the inlets and outlets)}. \tag{20}$$

Substitute and simplify,

$$0 = \dot{m}_{air}h_{air,in} - \dot{m}_{air}h_{air,out} + \dot{Q}_{added\ to\ CV}, \tag{21}$$

$$\dot{Q}_{added\ to\ CV} = \dot{m}_{air}\left(h_{air,out} - h_{air,in}\right). \tag{22}$$

Using the previously calculated values,

⇨ $\dot{Q}_{added\ to\ CV}$ = -577 kJ/min ($\boxed{\text{577 kJ/min leaves the air and goes into the R134a}}$)

Note that if we applied a control volume to just the R134a, we would find that 577 kJ/min enters the R134a from the air.