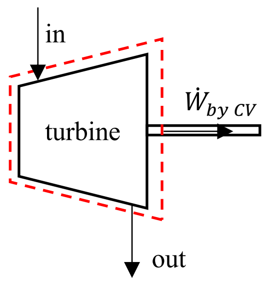
Air expands through a turbine from 8 bar (abs) and 960 K to 1 bar (abs) and 450 K. The inlet speed is small compared to the exit speed of 90 m/s. The turbine operates at steady state and develops a power output of 2500 kW. Heat transfer between the turbine and the surroundings is negligible.

Calculate the mass flow rate of air and the exit area.

SOLUTION:
Apply the First Law to a control volume surrounding the turbine.



$$\frac{dE_{CV}}{dt} = \sum_{in} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) - \sum_{out} \dot{m}\left(h + \tfrac{1}{2}V^2 + gz\right) + \dot{Q}_{into\ CV} - \dot{W}_{by\ CV,other}, \quad (1)$$

where,

$\frac{dE_{CV}}{dt} = 0$ (steady state assumed),

$V_{in}^2 \ll V_{out}^2$ (outlet speed >> inlet speed),

$gz_{in} \approx gz_{out} \approx 0$ (since dealing with a gas, changes in potential energy are negligible),

$\dot{Q}_{into\ CV} = 0$ (given that heat transfer is negligible),

$\dot{W}_{by\ CV,other} = 2500\ kW$ (given).

Also note that from conservation of mass,

$$\frac{dM_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}, \quad (2)$$

where,

$\frac{dM_{CV}}{dt} = 0$ (steady state assumed).

Thus,

$$\dot{m} = \dot{m}_{out} = \dot{m}_{in}. \quad (3)$$

Substituting into Eq. (1) and simplifying,

$$\dot{W}_{by\ CV,other} = \dot{m}(h_{in}) - \dot{m}\left(h_{out} + \tfrac{1}{2}V_{out}^2\right), \quad (4)$$

$$\dot{W}_{by\ CV,other} = \dot{m}\left(h_{in} - h_{out} - \tfrac{1}{2}V_{out}^2\right). \quad (5)$$

$$\dot{m} = \frac{\dot{W}_{by\ CV,other}}{h_{in} - h_{out} - \frac{1}{2}V_{out}^2}. \quad (6)$$

Using the given data,

h_{in} = 1001 kJ/kg (from the ideal gas table for air at 960 K),

h_{out} = 452.0 kJ/kg (from the ideal gas table for air at 450 K),

V_{out} = 90 m/s (given),

⇨ $\boxed{\dot{m} = 4.59 \text{ kg/s}}$.

Note: 1 kJ/kg = 1 (kN.m)/kg = 1000 (kg.m^2)/(kg.s^2) = 1000 m^2/s^2. This conversion is useful when evaluating the specific kinetic energy term.

To find the outlet area,

$$\dot{m} = \frac{V_{out}A_{out}}{v_{out}} \Rightarrow A_{out} = \frac{\dot{m}v_{out}}{V_{out}}, \quad (7)$$

where the ideal gas law can be used to determine the outlet specific volume,

$$v_{out} = \frac{R_{air}T_{out}}{p_{out}}. \quad (8)$$

Using the given parameters,

p_{out} = 1 bar (abs) = 100 kPa (abs),

R_{air} = 287 J/(kg.K),

T_{out} = 450 K,

⇨ $v_{out} = 1.292\ m^3/kg$

⇨ $\boxed{A_{out} = 0.0658\ m^2}$

Following is a Python code used for the calculations.

```
# COE_cv20.py

import numpy as np
# Import a class created for reading the Ideal Gas Tables.
from IdealGasTable import IdealGasTable

# Import the Ideal Gas Table data for air.
IGT = IdealGasTable("./IdealGasTables.xlsx", "Air")

T_in = 960  # K, inlet temperature, given
T_out = 450  # K, outlet temperature, given
h_in = IGT.GetTableValue('h', T=T_in) # kJ/kg, specific enthalpy at inlet
h_out = IGT.GetTableValue('h', T=T_out)  # kJ/kg, specific enthalpy at outlet
V_out = 90  # m/s, velocity at outlet, given
Wdot_by = 2500  # kW, power generated, given

# Calculate the mass flow rate.  Note the unit conversion.
mdot = Wdot_by/(h_in-h_out-0.5*(V_out**2)/1000)
print("h_in = %.3e kJ/kg" % h_in)
print("h_out = %.3e kJ/kg" % h_out)
print("mdot = %.3e kg/s" % mdot)

p_out = 1*100  # kPa, outlet pressure, given
R_air = 0.287  # kJ/(kg.K), gas constant for air
v_out = R_air*T_out/p_out  # Ideal Gas Law
print("v_out = %.3e m^3/kg" % v_out)

# Find the outlet area.
A_out = mdot*v_out/V_out
print("A_out = %.3e m^2" % A_out)
```

```
≫ python3 ./COE_cv20.py
h_in = 1.001e+03 kJ/kg
h_out = 4.520e+02 kJ/kg
mdot = 4.588e+00 kg/s
v_out = 1.291e+00 m^3/kg
A_out = 6.583e-02 m^2
```