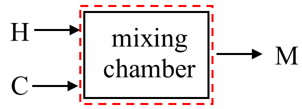
Consider an ordinary shower where hot water at 140 °F is mixed with cold water at 50 °F. If it is desired that a steady stream of warm water at 110 °F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.

SOLUTION:

Apply the First Law to the control volume shown below,



$$\frac{d}{dt}\int_{\text{CV}} e\rho\, dV + \int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{on}\\ \text{CV}}} \tag{1}$$

where,

$$\frac{d}{dt}\int_{\text{CV}} e\rho dV = 0 \quad \text{(steady flow)}$$

$$\int_{\text{CS}} \left(h + \tfrac{1}{2}V^2 + gz\right)\left(\rho \mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = -\dot{m}_H h_H - \dot{m}_C h_C + \dot{m}_M h_M$$

(Neglect changes in potential and kinetic energies since they won't be significant for flow through a shower head.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(negligible heat transfer to the surroundings} \Rightarrow \text{assume adiabatic)}$$

$$\dot{W}_{\substack{\text{on}\\ \text{CV}}} = 0 \quad \text{(no work besides pressure work is being done on the CV)}$$

Substitute,

$$-\dot{m}_H h_H - \dot{m}_C h_C + \dot{m}_M h_M = 0 \tag{2}$$

Apply conservation of mass to the same control volume to find,

$$\dot{m}_M = \dot{m}_H + \dot{m}_C \tag{3}$$

Combine Eqs. (2) and (3) and simplify,

$$-\dot{m}_H h_H - \dot{m}_C h_C + \left(\dot{m}_H + \dot{m}_C\right) h_M = 0$$

$$-\frac{\dot{m}_H}{\dot{m}_C} h_H - h_C + \left(\frac{\dot{m}_H}{\dot{m}_C} + 1\right) h_M = 0$$

$$\frac{\dot{m}_H}{\dot{m}_C}\left(h_M - h_H\right) = h_C - h_M$$

$$\boxed{\therefore \frac{\dot{m}_H}{\dot{m}_C} = \frac{h_C - h_M}{h_M - h_H}} \tag{4}$$

Look up the specific enthalpies for water in a thermodynamics reference (note that since the water is a pure liquid, the mixing pressure is irrelevant):

$h_C = h_{50\,°\text{F}} = 18.06\ \text{Btu/lb}_\text{m}$

$h_H = h_{140\,°\text{F}} = 107.96\ \text{Btu/lb}_\text{m}$

$h_M = h_{110\,°\text{F}} = 78.02\ \text{Btu/lb}_\text{m}$

$$\boxed{\therefore \frac{\dot{m}_H}{\dot{m}_C} = 2.0}$$