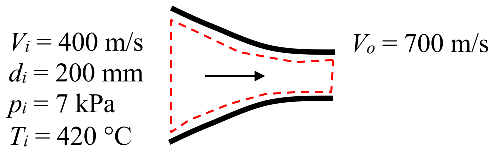
Air flows through a nozzle with an inlet diameter of 200 mm, velocity of 400 m/s, pressure of 7 kPa (abs), and temperature of 420 °C. The nozzle exit diameter is adjusted such that the exiting velocity is 700 m/s. Determine:

a. the exit temperature, and
b. the mass flow rate through the nozzle

SOLUTION:



Apply the First Law to the CV shown assuming 1D, steady flow, no heat transfer, and no "other" work done on the CV besides inlet/outlet pressure work,

$$\frac{d}{dt}\int_{CV} e\rho\, dV + \int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{Q}_{\substack{\text{into}\\ \text{CV}}} + \dot{W}_{\substack{\text{other,}\\ \text{on CV}}} \qquad (1)$$

where,

$$\frac{d}{dt}\int_{CV} e\rho dV = 0 \quad \text{(steady state)} \qquad (2)$$

$$\int_{CS}\left(h+\tfrac{1}{2}V^2+gz\right)\left(\rho\mathbf{u}_{\text{rel}}\cdot d\mathbf{A}\right) = \dot{m}\left(h_o+\tfrac{1}{2}V_o^2-h_i-\tfrac{1}{2}V_i^2\right) \qquad (3)$$

(The change in potential energies is assumed negligible and, since the flow is steady and 1D, the mass flow rate is the same at the inlet and outlet.)

$$\dot{Q}_{\substack{\text{into}\\ \text{CV}}} = 0 \quad \text{(Assume little heat transfer occurs over the nozzle surface area.)} \qquad (4)$$

$$\dot{W}_{\substack{\text{other,}\\ \text{on CV}}} = 0 \quad \text{(No work is performed other than inlet/outlet pressure work.)} \qquad (5)$$

Substitute and simplify,

$$\dot{m}\left(h_o+\tfrac{1}{2}V_o^2-h_i-\tfrac{1}{2}V_i^2\right)=0 \;\Rightarrow\; \boxed{h_o = h_i + \tfrac{1}{2}\left(V_i^2 - V_o^2\right)} \qquad (6)$$

For the given conditions,

- h_i = 705.75 kJ/kg (from thermodynamics tables for air, assumed to be an ideal gas, at T_i = 420 °C)
- V_i = 400 m/s
- V_o = 700 m/s
- $\Rightarrow$ h_o = 540.75 kJ/kg $\Rightarrow$ $\boxed{T_o = 264\ °\text{C}}$ (from thermo tables assuming air is an ideal gas)

The mass flow rate is,

$$\boxed{\dot{m} = \rho_i V_i A_i = \frac{p_i}{RT_i} V_i \tfrac{\pi}{4} d_i^2} \qquad (7)$$

Using the given parameters:

- p_i = 7 kPa (abs),
- R = 287 J/(kg·K),
- T_i = 420 °C = 693 K,
- => ρ_i = 0.03624 kg/m^3,
- V_i = 400 m/s,
- d_i = 0.2 m => A_i = 0.03142 m^2,
- $\Rightarrow$ $\boxed{\dot{m} = 0.442\ \text{kg/s}}$