i. Each line of the following table gives data for a process involving a closed system. Each entry has the same energy units. Determine the missing entries.

Process	$Q_{\text{into sys}}$	$W_{\text{by sys}}$	E_1	E_2	ΔE
a	+50		-20		+70
b		+20		+50	+30
c		-60	+40	+60	
d	-40		+50		0
e	+50	+150		-80	

ii. A gas contained within a piston-cylinder assembly undergoes two processes, A and B, between the same end states, 1 and 2, where p_1 =1 bar (abs), V_1 =1.0 m^3, U_1 =400 kJ and p_2 = 10 bar (abs), V_2 = 0.1 m^3, U_2 = 450 kJ.

- Process A: Constant-volume process from state 1 to a pressure of 10 bar (abs), followed by a constant-pressure process to state 2.
- Process B: Process from 1 to 2 during which the pressure-volume relation is pV = constant.

Kinetic and potential energy effects can be ignored. For each of the processes A and B,

a. Sketch the process on a *p*-*V* diagram.
b. Evaluate the work, in kJ.
c. Evaluate the heat transfer, in kJ.

SOLUTION:

i. Apply the First Law to the system for each process.

Process **a**: $\Delta E_{sys} = E_2 - E_1 = Q_{into\ sys} - W_{by\ sys}$, (1)

Using the given values,

ΔE_{sys} = +70, E_1 = -20 => $\boxed{E_2 = +50}$,

$Q_{\text{into sys}}$ = +50 => $\boxed{W_{\text{by sys}} = -20}$

Process **b**: $\Delta E_{sys} = E_2 - E_1 = Q_{into\ sys} - W_{by\ sys}$, (2)

Using the given values,

ΔE_{sys} = +30, E_2 = +50 => $\boxed{E_1 = +20}$,

$W_{\text{by sys}}$ = +20 => $\boxed{Q_{\text{into sys}} = +50}$

Process **c**: $\Delta E_{sys} = E_2 - E_1 = Q_{into\ sys} - W_{by\ sys}$, (3)

Using the given values,

E_1 = +40, E_2 = +60 => $\boxed{\Delta E = +20}$,

$W_{\text{by sys}}$ = -60 => $\boxed{Q_{\text{into sys}} = -40}$

Process **d**: $\Delta E_{sys} = E_2 - E_1 = Q_{into\ sys} - W_{by\ sys}$, (4)

Using the given values,

E_1 = +50, ΔE_{sys} = 0=> $\boxed{E_2 = +50}$,

$Q_{\text{into sys}}$ = -40 => $\boxed{W_{\text{by sys}} = -40}$

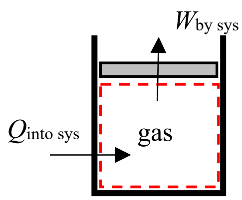
Process **e**: $\Delta E_{sys} = E_2 - E_1 = Q_{into\ sys} - W_{by\ sys}$, (5)

Using the given values,

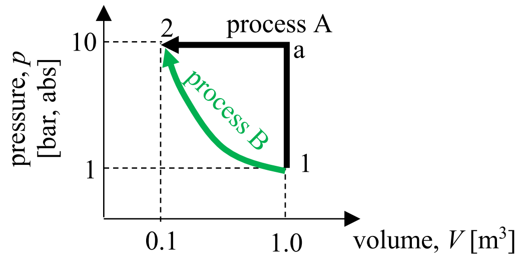
$Q_{\text{into sys}}$ = +50, $W_{\text{by sys}}$ = +150 => $\boxed{\Delta E = -100}$,

E_2 = -80, ΔE_{sys} = -100=> $\boxed{E_1 = +20}$

ii.



The two processes are sketched on the following *p*-*V* plot.



First evaluate the (boundary) work for process A,

$$W_{by\ sys,A} = \underbrace{\int_{V_1}^{V_a} p dV}_{=0 \text{ since } V_a = V_1} + \underbrace{\int_{V_a}^{V_2} p dV}_{\substack{=p_2(V_2 - V_a) \\ \text{since } p = \text{constant}}} = p_2(V_2 - V_a). \quad (6)$$

Using the given values,

p_2 = 10 bar (abs) = 10*10^5 Pa (abs),
V_2 = 0.1 m^3,
V_1 = 1.0 m^3,
⇨ $\boxed{W_{by\ sys,A} = -900 \text{ kJ}}$.

Note that this work is equal to the area under the curve (process A) in the *p*-*V* plot. The negative sign occurs because work is done on the system (the gas is getting compressed to a smaller volume).

Similarly, the (boundary) work for process B is,

$$W_{by\ sys,B} = \int_{V_1}^{V_2} p dV = \underbrace{\int_{V_1}^{V_2} \frac{c}{V} dV}_{\text{since } pV = c} = c\ln\left(\frac{V_2}{V_1}\right), \quad (7)$$

where the constant can be found using one of the states. For example,

$$p_1 V_1 = c = p_2 V_2, \quad (8)$$

Substituting Eq. (8) into Eq. (7) and using state 1 to find the constant,

$$W_{by\ sys,B} = p_1 V_1 \ln\left(\frac{V_2}{V_1}\right). \quad (9)$$

Using the given values,

p_1 = 1 bar (abs) = 1*10^5 Pa (abs),
V_2 = 0.1 m^3,
V_1 = 1.0 m^3,
⇨ c = 1*10^5 J,
⇨ $\boxed{W_{by\ sys,B} = -230 \text{ kJ}}$.

Note that the work for process B is different than the work for process A since a different path is taken.

To determine the energy transferred into the system via heat transfer, apply the First Law to the gas (the system),

$$\Delta E_{sys} = Q_{into\ sys} - W_{by\ sys}, \tag{10}$$

where,

$$\Delta E_{sys} = \Delta U_{sys} + \Delta KE_{sys} + \Delta PE_{sys} \approx \Delta U_{sys}$$ (since ΔKE and ΔPE are negligible).

Substituting and re-arranging,

$$Q_{into\ sys} = \Delta U_{sys} + W_{by\ sys}. \tag{11}$$

Using the given and previously calculated values for process A,

$\Delta U_{sys} = U_2 - U_1 = 450\text{ kJ} - 400\text{ kJ} = 50\text{ kJ}$,
$W_{by\ sys,A} = -900\text{ kJ}$,
⇨ $\boxed{Q_{into\ sys,A} = -850\text{ kJ}}$.

For process B,

$\Delta U_{sys} = U_2 - U_1 = 450\text{ kJ} - 400\text{ kJ} = 50\text{ kJ}$,
$W_{by\ sys,B} = -230\text{ kJ}$,
⇨ $\boxed{Q_{into\ sys,B} = -180\text{ kJ}}$.

Note that the change in internal energy is independent of the process since it is a property, i.e., it only depends on states 1 and 2, not the path between them. Heat transfer and work are path dependent (and not properties), which is why the work and heat transfer for processes A and B are different.