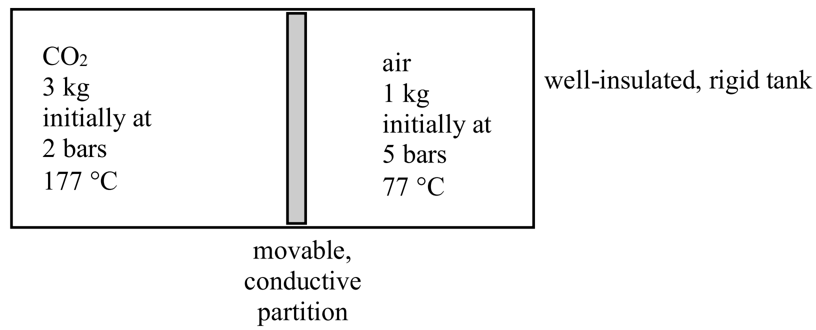
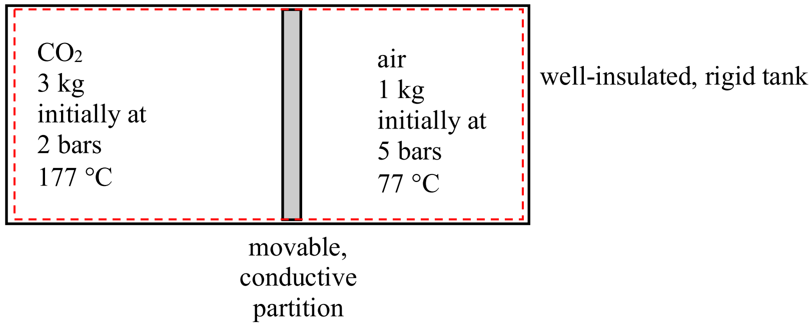
One kilogram of air, initially at 5 bars (abs) and 77 °C, and 3 kg of carbon dioxide (CO_2), initially at 2 bars (abs) and 177 °C, are confined to opposite sides of a rigid, well-insulated container. The partition is free to move and allows conduction from one gas to the other without energy storage in the partition itself.
Determine:
a) the final equilibrium temperature
b) and the final pressure
You may assume that the specific heats for both the air and CO_2 remain constant over the range of temperatures: $c_{v,air}$=0.726 kJ/(kg·K); $c_{v,CO2}$=0.750 kJ/(kg·K); $c_{p,air}$=1.013 kJ/(kg·K); $c_{p,CO2}$=0.939 kJ/(kg·K)



SOLUTION:

Apply the 1st Law to the following CV.



$$\Delta E_{\text{CV}} = Q_{\substack{\text{into}\\\text{CV}}} + W_{\substack{\text{on}\\\text{CV}}} \tag{1}$$

where

$$\Delta E_{\text{CV}} = \Delta U_{\text{CV}} = U_f - U_i = m_{\text{CO}_2}\left(u_{\text{CO}_2,f} - u_{\text{CO}_2,i}\right) + m_{\text{air}}\left(u_{\text{air},f} - u_{\text{air},i}\right)$$

$$Q_{\substack{\text{into}\\\text{CV}}} = 0 \quad \text{(well-insulated tank)}$$

$$W_{\substack{\text{on}\\\text{CV}}} = 0 \quad \text{(rigid tank)}$$

Assuming constant specific heats (perfect gases) and simplifying COE gives:

$$m_{\text{CO}_2} c_{v,\text{CO}_2}\left(T_{\text{CO}_2,f} - T_{\text{CO}_2,i}\right) + m_{\text{air}} c_{v,\text{air}}\left(T_{\text{air},f} - T_{\text{air},i}\right) = 0$$

Since the partition is conductive, $T_{\text{CO}_2,f} = T_{\text{air},f} = T_f$ resulting in:

$$m_{\text{CO}_2} c_{v,\text{CO}_2}\left(T_f - T_{\text{CO}_2,i}\right) + m_{\text{air}} c_{v,\text{air}}\left(T_f - T_{\text{air},i}\right) = 0$$

$$\boxed{\therefore T_f = \frac{m_{\text{CO}_2} c_{v,\text{CO}_2} T_{\text{CO}_2,i} + m_{\text{air}} c_{v,\text{air}} T_{\text{air},i}}{m_{\text{CO}_2} c_{v,\text{CO}_2} + m_{\text{air}} c_{v,\text{air}}}} \tag{2}$$

Using the given values:

m_{CO2} = 3 kg
$c_{v,\text{CO2}}$ = 0.750 kJ/(kg·K)
$T_{\text{CO2},i}$ = 177 °C = 450 K
m_{air} = 1 kg
$c_{v,\text{air}}$ = 0.726 kJ/(kg·K)
$T_{\text{air},i}$ = 77 °C = 350 K
$\Rightarrow$ $\boxed{T_f = 426 \text{ K} = 153\ °\text{C}}$

The final pressure in each compartment will be the same otherwise the partition would continue to move. Use the ideal gas law to determine the final densities of the gases in terms of the final temperature and pressure.

$$p = \rho RT \Rightarrow \rho = \frac{p}{RT} \tag{3}$$

In addition, the total volume of the tank is the sum of the final volumes occupied by each gas.

$$V_{\text{tank}} = \underbrace{\frac{m_{\text{CO}_2}}{\rho_{\text{CO}_2,f}}}_{=V_{\text{CO}_2,f}} + \underbrace{\frac{m_{\text{air}}}{\rho_{\text{air},f}}}_{=V_{\text{air},f}} \tag{4}$$

Combine Eqns. (3) and (4) and simplify.

$$V_{\text{tank}} = m_{\text{CO}_2}\frac{R_{\text{CO}_2}T_f}{p_f} + m_{\text{air}}\frac{R_{\text{air}}T_f}{p_f}$$

$$p_f = m_{\text{CO}_2}\frac{R_{\text{CO}_2}T_f}{V_{\text{tank}}} + m_{\text{air}}\frac{R_{\text{air}}T_f}{V_{\text{tank}}}$$

$$\boxed{p_f = \frac{T_f}{V_{\text{tank}}}\left(m_{\text{CO}_2}R_{\text{CO}_2} + m_{\text{air}}R_{\text{air}}\right)} \tag{5}$$

The tank volume is known from the initial masses, pressures, and temperatures.

$$\boxed{V_{\text{tank}} = \underbrace{\frac{m_{\text{CO}_2}}{\rho_{\text{CO}_2,i}}}_{=V_{\text{CO}_2,i}} + \underbrace{\frac{m_{\text{air}}}{\rho_{\text{air},i}}}_{=V_{\text{air},i}} = m_{\text{CO}_2}\frac{R_{\text{CO}_2}T_{\text{CO}_2,i}}{p_{\text{CO}_2,i}} + m_{\text{air}}\frac{R_{\text{air}}T_{\text{air},i}}{p_{\text{air},i}}} \tag{6}$$

Using the given data:

m_{CO2} = 3 kg
R_{CO2} = 0.1889 kJ/(kg·K)
$T_{CO2,i}$ = 177 °C = 450 K
$p_{CO2,i}$ = 2 bar = 202.7 kPa
m_{air} = 1 kg
R_{air} = 0.287 kJ/(kg·K)
$T_{air,i}$ = 77 °C = 350 K
$p_{air,i}$ = 5 bar = 506.63 kPa
T_f = 426 K (from previous part of the problem)
⇒ $\boxed{V_{\text{tank}} = 1.46 \text{ m}^3}$ and $\boxed{p_f = 250 \text{ kPa} = 2.46 \text{ bar}}$