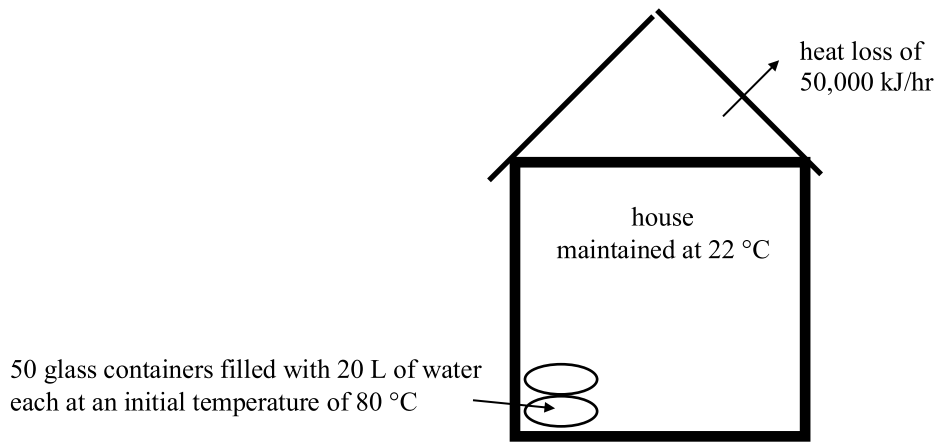
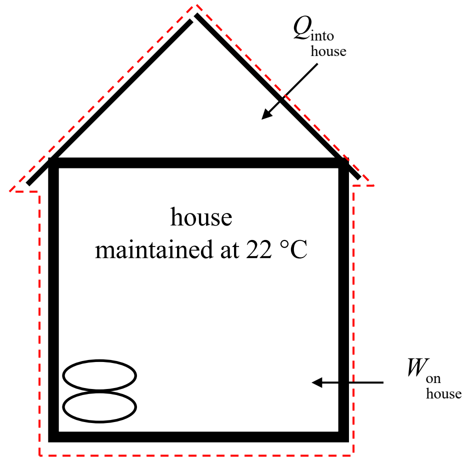
A passive solar house that is losing heat to the outdoors at an average rate of 50,000 kJ/hr is maintained at 22 °C at all times during a winter night for 10 hr. The house is to be heated by 50 glass containers each containing 20 L of water that is heated to 80 °C during the day by absorbing solar energy. A thermostat-controlled, 15 kW back-up electric resistance heater turns on whenever necessary to keep the house at 22 °C.

a. How long will the electric heating system need to run during the night?
b. How long would the electric heater run during the night if the house did not incorporate solar heating?



SOLUTION:

Apply the 1st Law to the house.



$$\Delta E_{\text{system}} = Q_{\substack{\text{into}\\\text{system}}} + W_{\substack{\text{on}\\\text{system}}} \tag{1}$$

The change in total energy of the house will consist of the change in the internal energy (potential and kinetic energy changes will be negligible). Furthermore, the total internal energy change will include the total energy change in the house structure, house air, and water tanks.

$$\Delta E_{\text{system}} = \Delta U_{\text{system}} = \Delta U_{\substack{\text{house}\\\text{structure}}} + \Delta U_{\text{air}} + \Delta U_{\text{water}} \tag{2}$$

Since the house structure and air are maintained at a constant temperature, $\Delta U_{\text{house}} = \Delta U_{\text{air}} = 0$. Hence, Eqn. (1) can be re-written as:

$$\Delta U_{\text{water}} = Q_{\substack{\text{into}\\\text{system}}} + W_{\substack{\text{on}\\\text{system}}} \tag{3}$$

The total change in the internal energy of the water (assuming an incompressible fluid) is given by:

$$\Delta U_{\text{water}} = m_{\text{water}} c_{\text{water}} \left(T_{f,\text{water}} - T_{i,\text{water}}\right) \tag{4}$$

The total heat added to the house is:

$$Q_{\substack{\text{into}\\\text{system}}} = \left(-50,000 \text{ kJ/hr}\right)\left(10 \text{ hr}\right) = -500,000 \text{ kJ} \tag{5}$$

and the total work done on the house by the electric heater is:

$$W_{\substack{\text{on}\\\text{system}}} = \left(15 \text{ kW}\right)\Delta t \tag{6}$$

where Δt is the time over which the heater operates.

Substitute Eqns. (4)-(6) into Eqn. (3).

$$m_{\text{water}} c_{\text{water}} \left(T_{f,\text{water}} - T_{i,\text{water}}\right) = -500,000 \text{ kJ} + \left(15 \text{ kW}\right)\Delta t \tag{7}$$

Using the given parameters in Eqn. (7).

m_{water} = 50(20 L)(0.001 m^3/L)(1000 kg/m^3) = 1000 kg
c_{water} = 4.179 kJ/(kg·K) (from a thermodynamics table)
$T_{f,\text{water}}$ = 22 °C
$T_{i,\text{water}}$ = 80 °C
$\Rightarrow$ $\boxed{\Delta t = 4.8 \text{ hrs}}$ Hence, the heater must be on for 4.8 hrs at night with the water tanks.

If the water containers were not present, then the left-hand side of Eqn (7) would be zero ($\Delta U_{\text{water}} = 0$) and:

$\Rightarrow$ $\boxed{\Delta t = 9.3 \text{ hrs}}$ Hence, the heater must be on for 9.3 hrs at night without the water tanks.