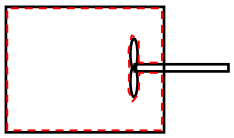
A gas is contained in a closed rigid tank fitted with a paddle wheel. The paddle wheel stirs the gas for 20 min, with the power varying with time t according to (10 W/min)t. Heat transfer from the gas to the surroundings takes place at a constant rate of 50 W. Determine:

a. the rate of change of energy of the gas at time 10 min, in watts, and
b. the net change in energy of the gas after 20 min, in kJ.

SOLUTION:

Apply the First Law to the system of gas within the tank,



$$\frac{dE_{\text{sys}}}{dt} = \dot{Q}_{\substack{\text{added}\\ \text{to sys}}} + \dot{W}_{\text{on sys}}\,, \tag{1}$$

where $\dot{Q}_{\text{into sys}} = -50\ \text{W}$ and $\dot{W}_{\text{on sys}} = \left(10\ {}^{\text{W}}\!/_{\text{min}}\right)t$. Thus, at t = 10 min,

$$\frac{dE_{\text{sys}}}{dt} = -50\ \text{W} + \left(10\ {}^{\text{W}}\!/_{\text{min}}\right)t\,, \tag{2}$$

$$\boxed{\therefore \frac{dE_{\text{sys}}}{dt} = 50\ \text{W}}.$$

The net change in energy of the gas is found by integrating Eqn. (2) in time,

$$\Delta E_{\text{sys}} = \int_{t=0}^{t=20\ \text{min}} \frac{dE_{\text{sys}}}{dt}dt = \int_{t=0}^{t=20\ \text{min}} \left[50\ \text{W} + \left(10\ {}^{\text{W}}\!/_{\text{min}}\right)t\right]dt = \left[\left(50\ \text{W}\right)t + \tfrac{1}{2}\left(10\ {}^{\text{W}}\!/_{\text{min}}\right)t^2\right]_{t=0}^{t=20\ \text{min}} \tag{3}$$

$$\boxed{\therefore \Delta E_{\text{sys}} = 60\ \text{kJ}}$$