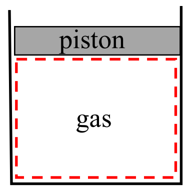
A gas in a piston assembly undergoes a polytropic expansion from an initial volume, $V_i$=0.1 m$^3$, and initial pressure, $p_i$ = 2 bar (abs) (1 bar = 1*10$^5$ Pa), to a final volume of $V_f$ = 0.5 m$^3$. Determine the work the gas does on the piston for $n$ = 1.5 and $n$ = 1 (where $pV^n$ = constant).

SOLUTION:

The work the gas performs on the piston is given by:




$$W_{i\to f} = \int_{V=0.1\text{ m}^3}^{V=0.5\text{ m}^3} p\,dV \tag{1}$$

where, for a polytropic expansion,

$$pV^n = \text{constant} = c \tag{2}$$

where $n$ is a constant. Substitute Eq. (2) into Eq. (1).

$$W_{i\to f} = \int_{V=0.1\text{ m}^3}^{V=0.5\text{ m}^3} cV^{-n}dV = \begin{cases} \dfrac{c}{1-n}V^{1-n}\Big|_{0.1\text{ m}^3}^{0.5\text{ m}^3} & n \neq 1 \\ c\ln V\Big|_{0.1\text{ m}^3}^{0.5\text{ m}^3} & n = 1 \end{cases} \tag{3}$$

When $n$ = 1.5, the constant is

$$c = \Big(\underbrace{2 * 10^5\text{ Pa}}_{=p_i}\Big)\Big(\underbrace{0.1\text{ m}^3}_{=V_i}\Big)^{1.5} = 6.32 * 10^3\text{ N}\cdot\text{m}^{2.5} \tag{4}$$

and the work performed by the gas, using Eq. (3), is:

$$W_{i\to f} = \frac{6.32*10^3\text{ N}\cdot\text{m}^{2.5}}{-0.5}\left[(0.5\text{ m}^3)^{-0.5} - (0.1\text{ m}^3)^{-0.5}\right], \tag{5}$$

$$\boxed{W_{i\to f} = 2.2 * 10^4\text{ N}\cdot\text{m}}. \tag{6}$$

When $n$ = 1, the constant is:

$$c = \Big(\underbrace{2 * 10^5\text{ Pa}}_{=p_i}\Big)\Big(\underbrace{0.1\text{ m}^3}_{=V_i}\Big) = 2 * 10^4\text{ N}\cdot\text{m} \tag{7}$$

and the work performed by the gas, using Eq. (3), is:

$$W_{i\to f} = (2 * 10^4\text{ N}\cdot\text{m})\ln\left(\frac{0.5\text{ m}^3}{0.1\text{ m}^3}\right), \tag{8}$$

$$\boxed{W_{i\to f} = 3.2 * 10^4\text{ N}\cdot\text{m}}. \tag{9}$$