

Combinational Logic Gates in CMOS

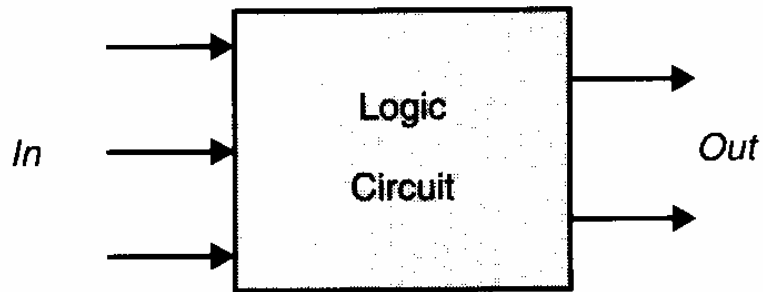
References:

Adapted from: *Digital Integrated Circuits: A Design Perspective*, J. Rabaey, Prentice Hall © UCB

Principles of CMOS VLSI Design: A Systems Perspective,
N. H. E. Weste, K. Eshraghian, Addison Wesley

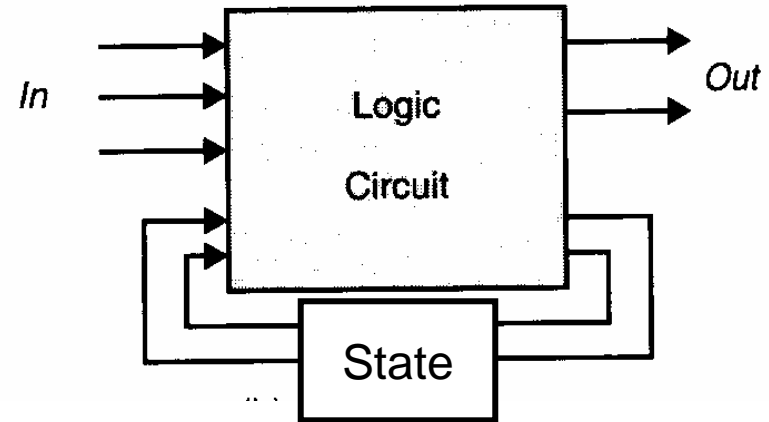
Adapted from: EE216A Lecture Notes by Prof. K. Bult ©
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Combinational vs. Sequential Logic



Combinational

$$\text{Out} = f(\text{In})$$



Sequential

$$\text{Out} = f(\text{In}, \text{State})$$

State is related to previous inputs
Stored in registers, memory etc

Overview

- Static CMOS
 - Complementary CMOS
 - Ratioed Logic
 - Pass Transistor/Transmission Gate Logic
- Dynamic CMOS Logic
 - Domino
 - np-CMOS

Static CMOS Circuit

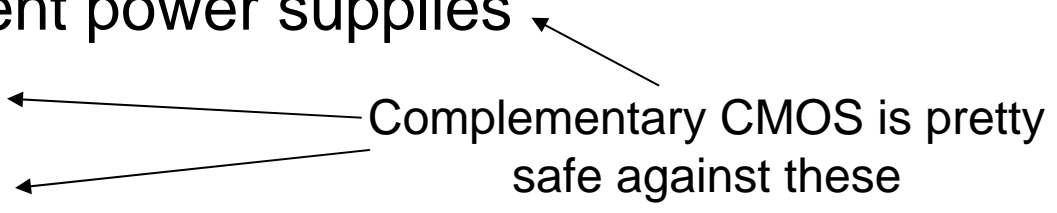
- At every point in time (except during the switching transients) each gate output is connected to either V_{DD} or V_{SS} via a low-resistive path
- The outputs of the gates assume at all times the value of the Boolean function, implemented by the circuit
- In contrast, a dynamic circuit relies on temporary storage of signal values on the capacitance of high impedance circuit nodes

Digital Gates

Fundamental Parameters

- Area and Complexity
- Performance
- Power Consumption
- Robustness and Reliability

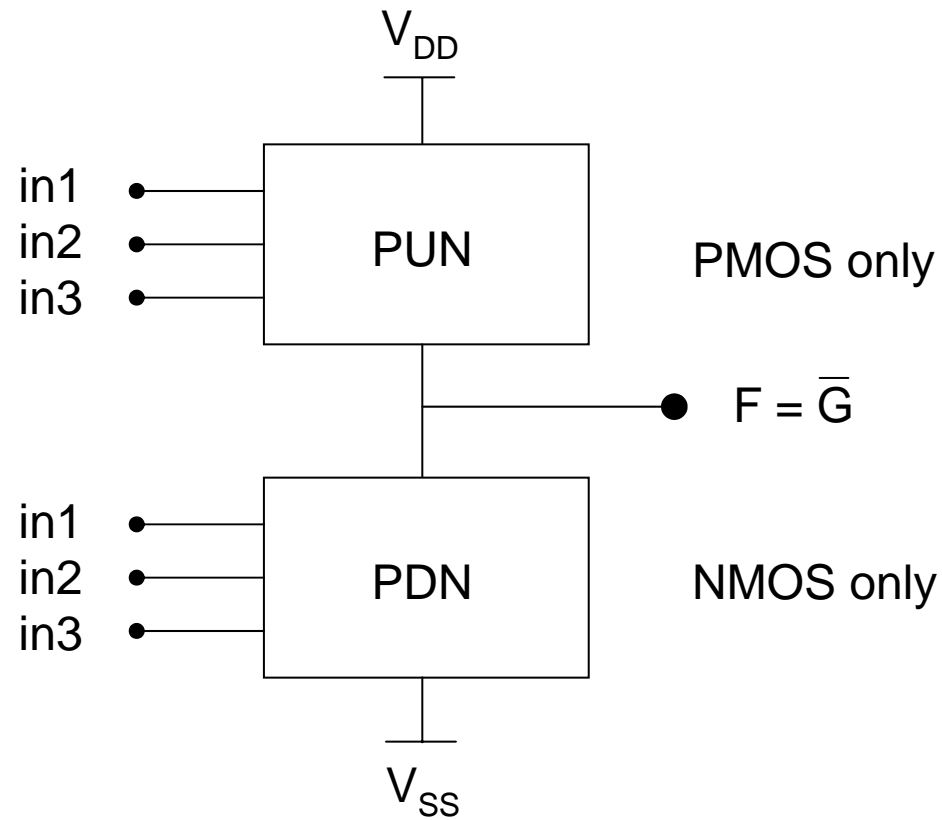
What Can Go Wrong in CMOS Logic?

- Incorrect or insufficient power supplies
 - Power supply noise
 - Noise on gate input
 - Faulty connections between transistors
 - Clock frequency too high or circuit too slow
- Complementary CMOS is pretty safe against these
- 

How about Ratioed or Dynamic Logic?

- All the previous and
- Incorrect ratios in ratioed logic
- Charge sharing in dynamic logic
- Incorrect clocking in dynamic logic

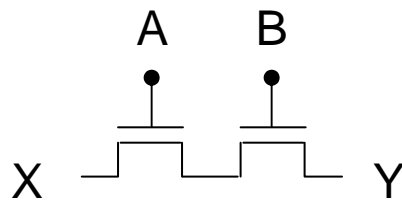
Complementary CMOS



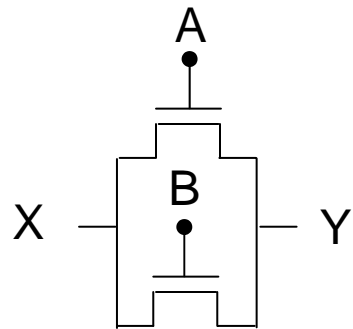
PUN and PDN are dual networks

NMOS Transistors in Series/Parallel Connection

- Transistors can be thought as a switch controlled by its gate signal
- NMOS switch closes when switch control input is high



$X = Y$ if $A = 1$ and $B = 1$, i.e., $AB = 1$

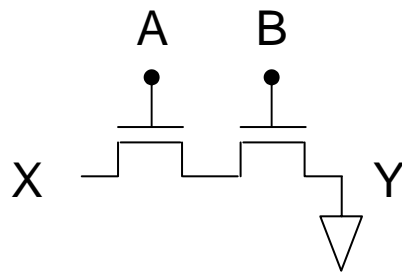


$X = Y$ if $A = 1$ or $B = 1$, i.e., $A + B = 1$

- NMOS passes a strong 0 but a weak 1

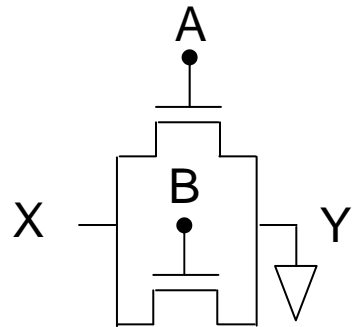
NMOS Transistors in Series/Parallel Connection

- Connect Y to GND



$X = 0$ if $A = 1$ and $B = 1$, i.e., $A.B = 1$

$$X = \overline{A.B}$$



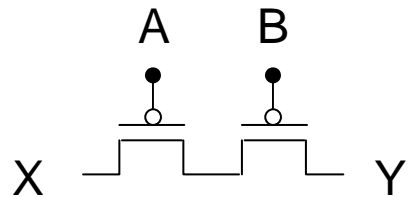
$X = 0$ if $A = 1$ or $B = 1$, i.e., $A + B = 1$

$$X = \overline{A + B}$$

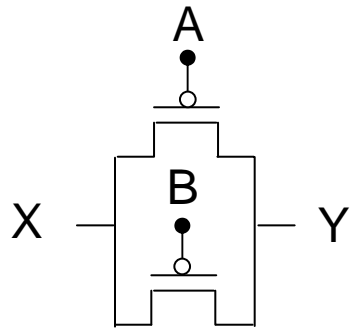
- Implement the complement of PDN

PMOS Transistors in Series/Parallel Connection

- PMOS switch closes when switch control input is low



$$X = Y \text{ if } \overline{A = 0 \text{ and } B = 0}$$
$$\text{or } \overline{A + B} = 1$$
$$\text{or } A.B = 1$$

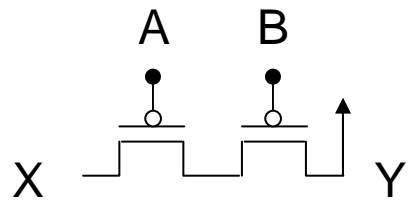


$$X = Y \text{ if } \overline{A = 0 \text{ or } B = 0}$$
$$\overline{A.B} = 1$$
$$\overline{A} + \overline{B} = 1$$

- PMOS passes a strong 1 but a weak 0

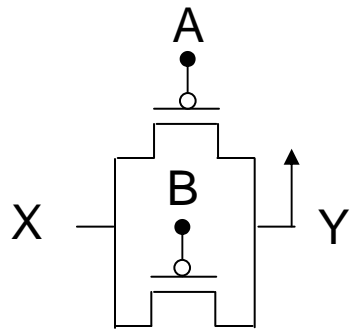
PMOS Transistors in Series/Parallel Connection

- Connect Y to VDD



$X = 1$ if $A = 0$ and $B = 0$

$$X = \overline{A + B} = \overline{A} \cdot \overline{B}$$



$X = 1$ if $A = 0$ or $B = 0$

$$X = \overline{A \cdot B} = \overline{A} + \overline{B}$$

- Combine series PDN and parallel PUN or parallel PDN and series PUN to complete the logic design to output good 1 and 0

Complementary CMOS Logic Style Construction

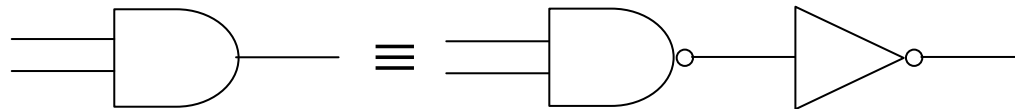
- PUN is the DUAL of PDN (can be shown using DeMorgan's Theorems)

$$\overline{A + B} = \overline{A} \overline{B}$$

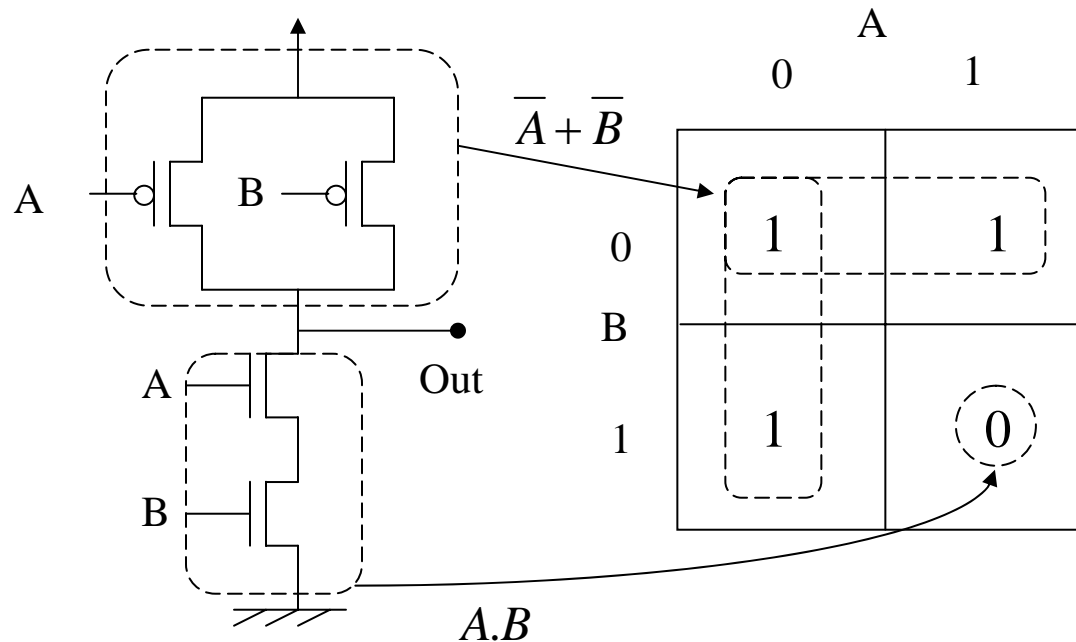
$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{G(in_1, in_2, in_3, \dots)} \equiv F(\overline{in_1}, \overline{in_2}, \overline{in_3}, \dots)$$

- The complementary gate is inverting
 - Implements NAND, NOR, ...
 - Non-inverting boolean function needs an inverter



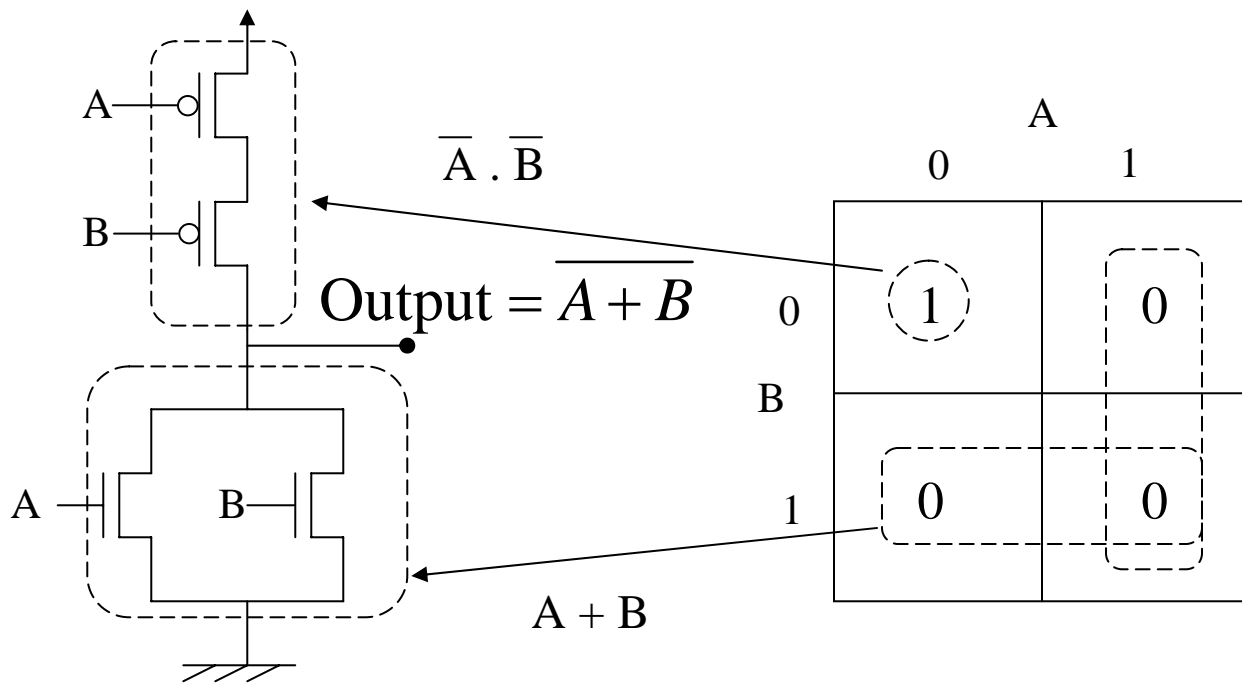
The NAND Circuit



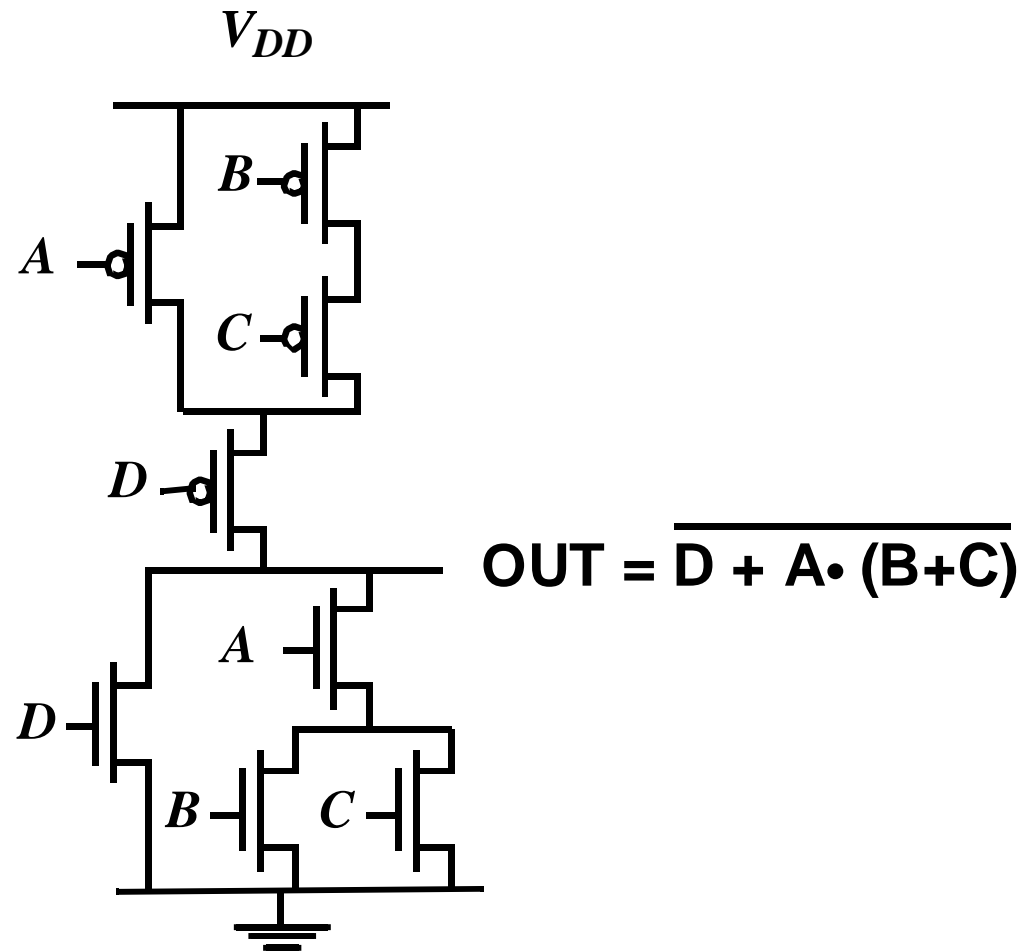
PDN connected to GND : $\overline{G} = \overline{A.B}$
 PUN connected to V_{DD} : $F = \overline{A} + \overline{B} = \overline{AB}$

$G(in_1, in_2, in_3, \dots) \equiv F(\overline{in_1}, \overline{in_2}, \overline{in_3}, \dots)$

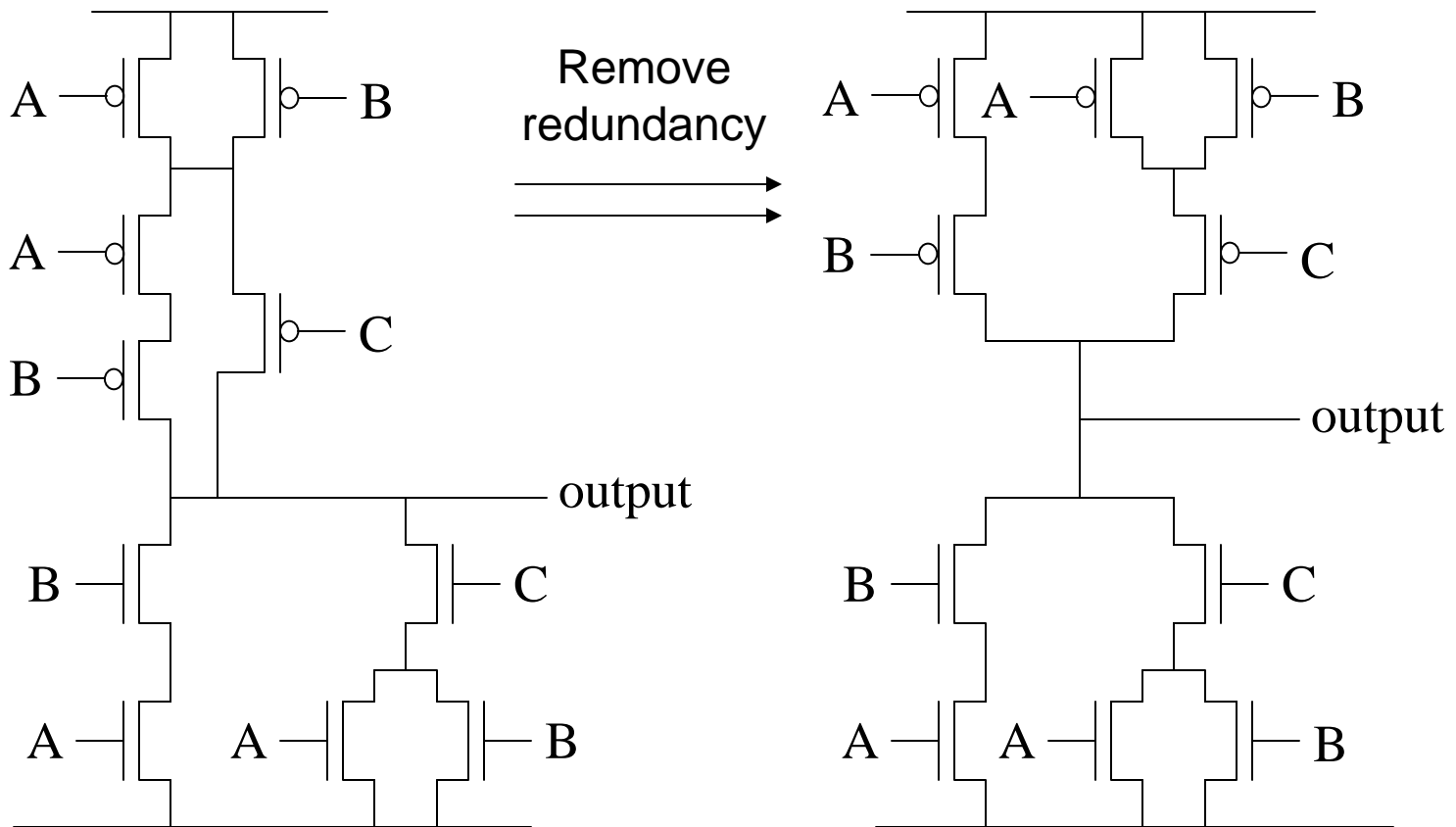
The NOR Circuit



Example Gate: COMPLEX CMOS GATE

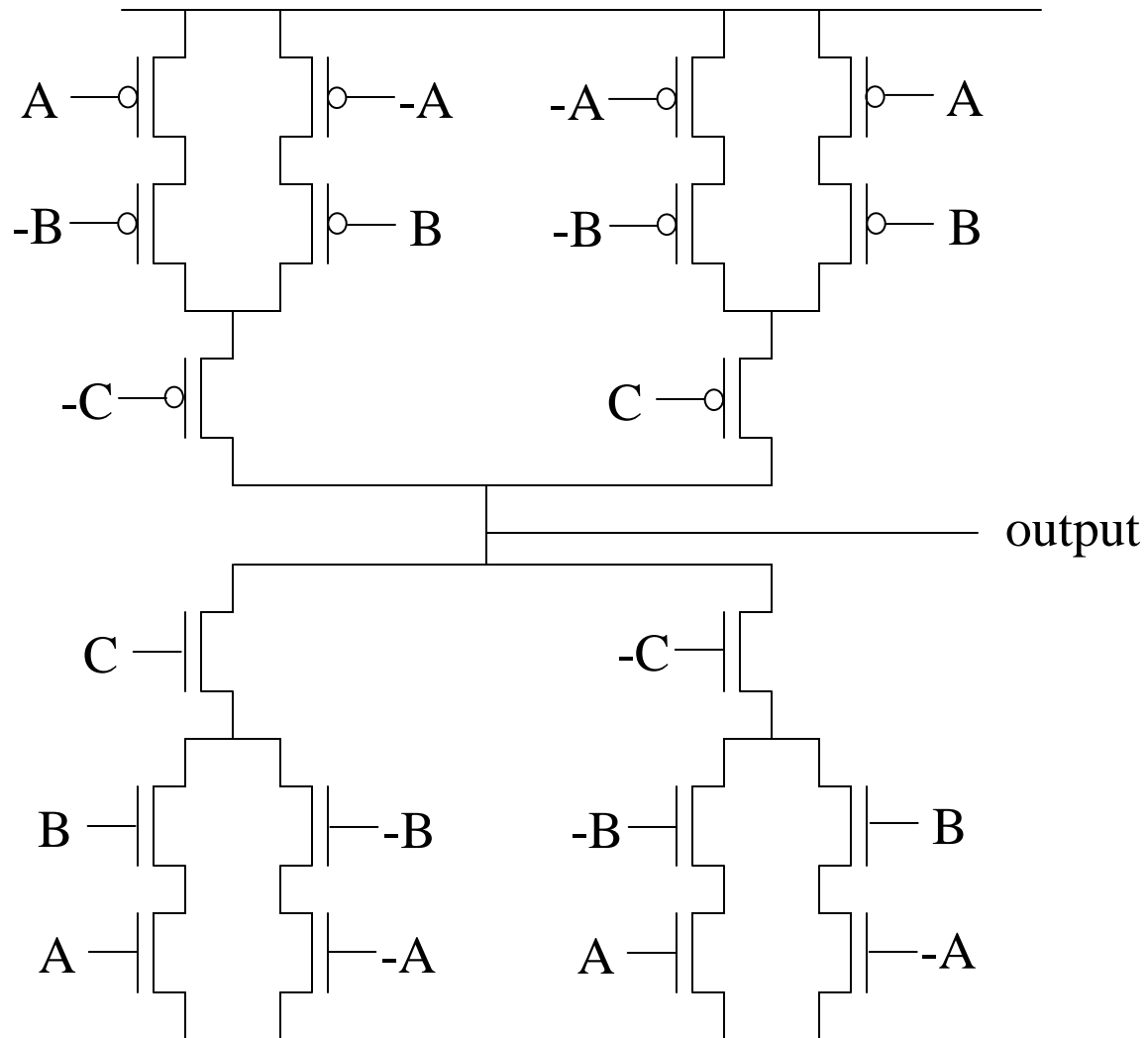


$$F = \overline{((A.B) + C.(A+B))} = \overline{\text{carry}}$$

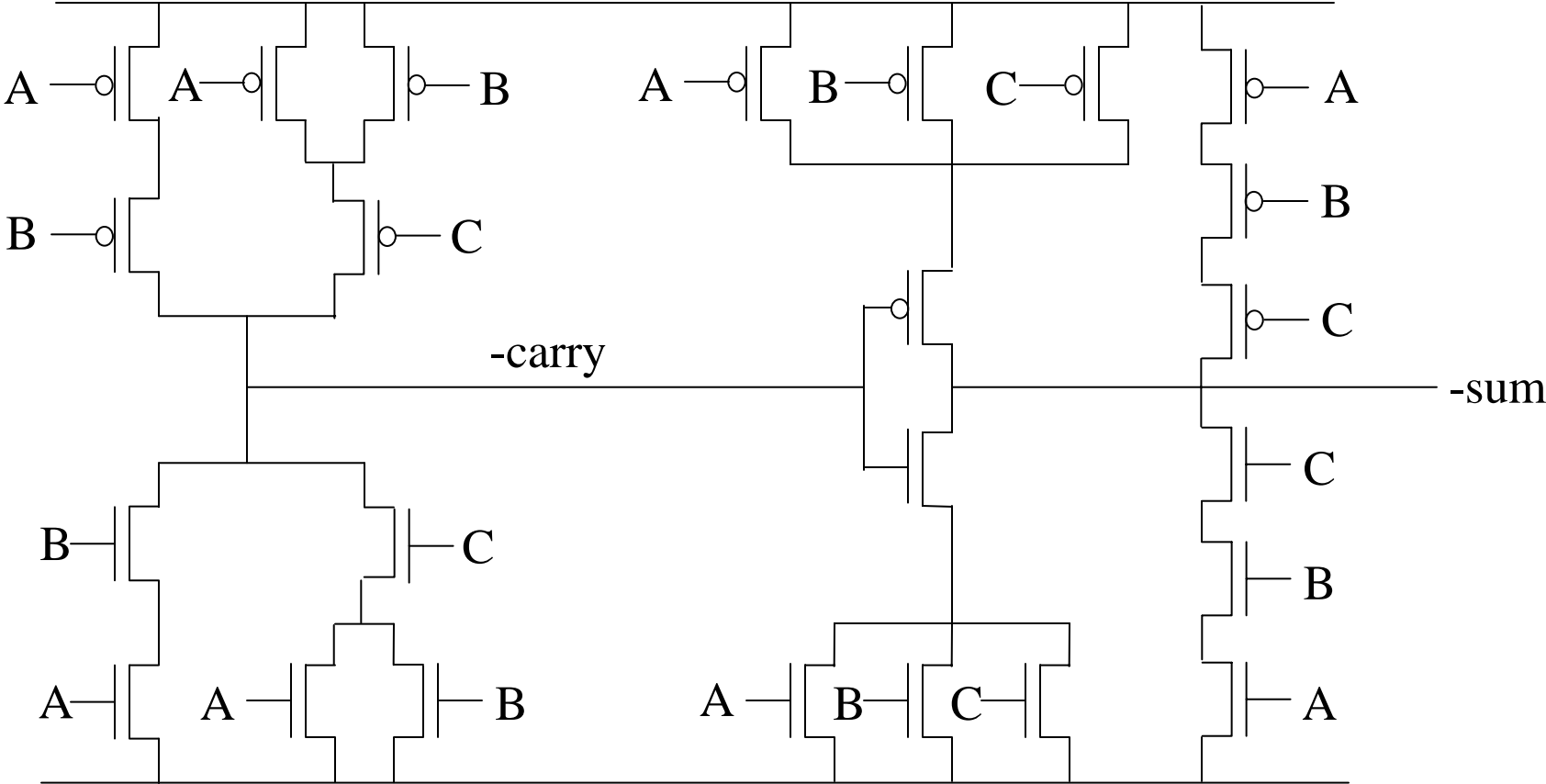


Symmetrical !

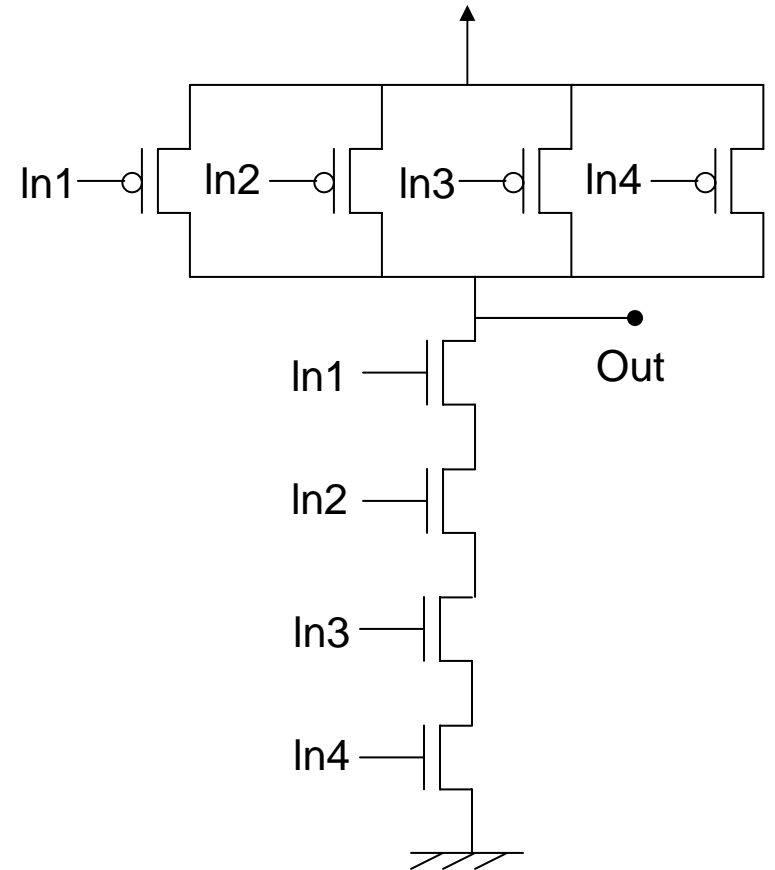
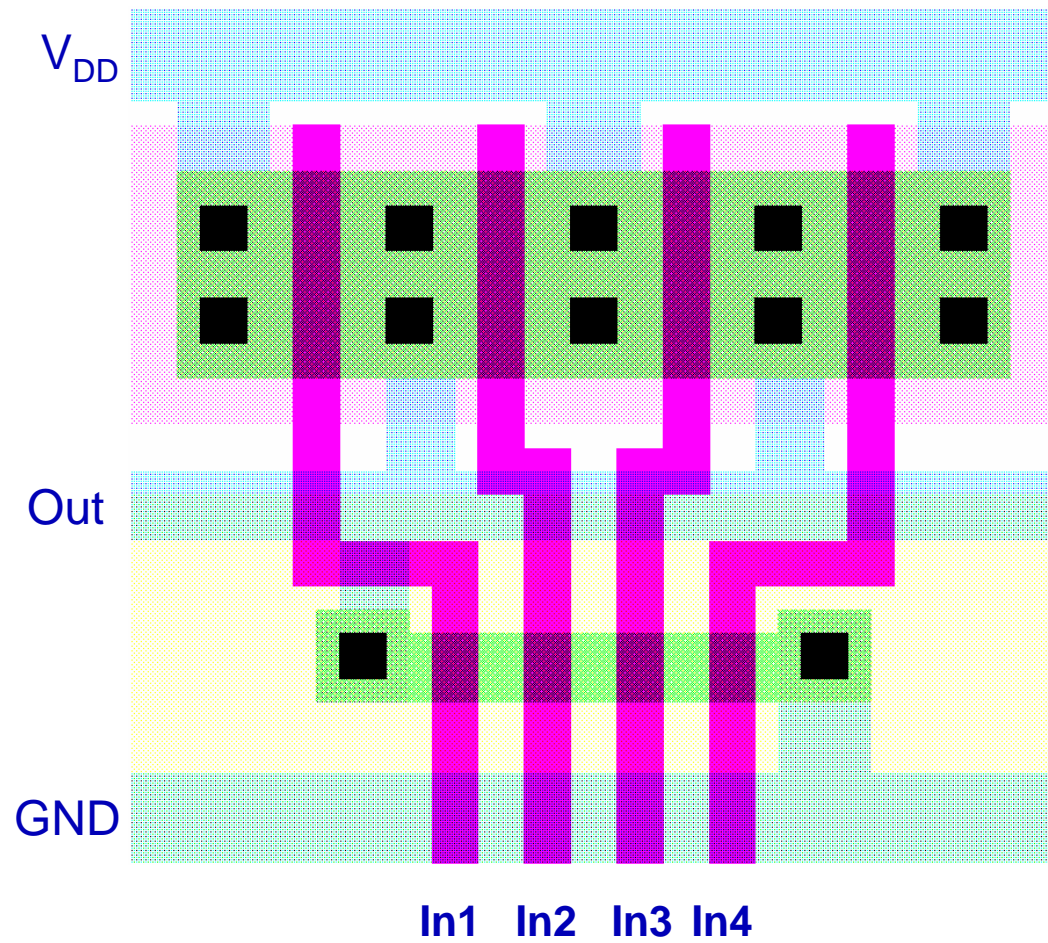
$$F = \overline{(ABC + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}\bar{C})} = \overline{\text{sum}}$$



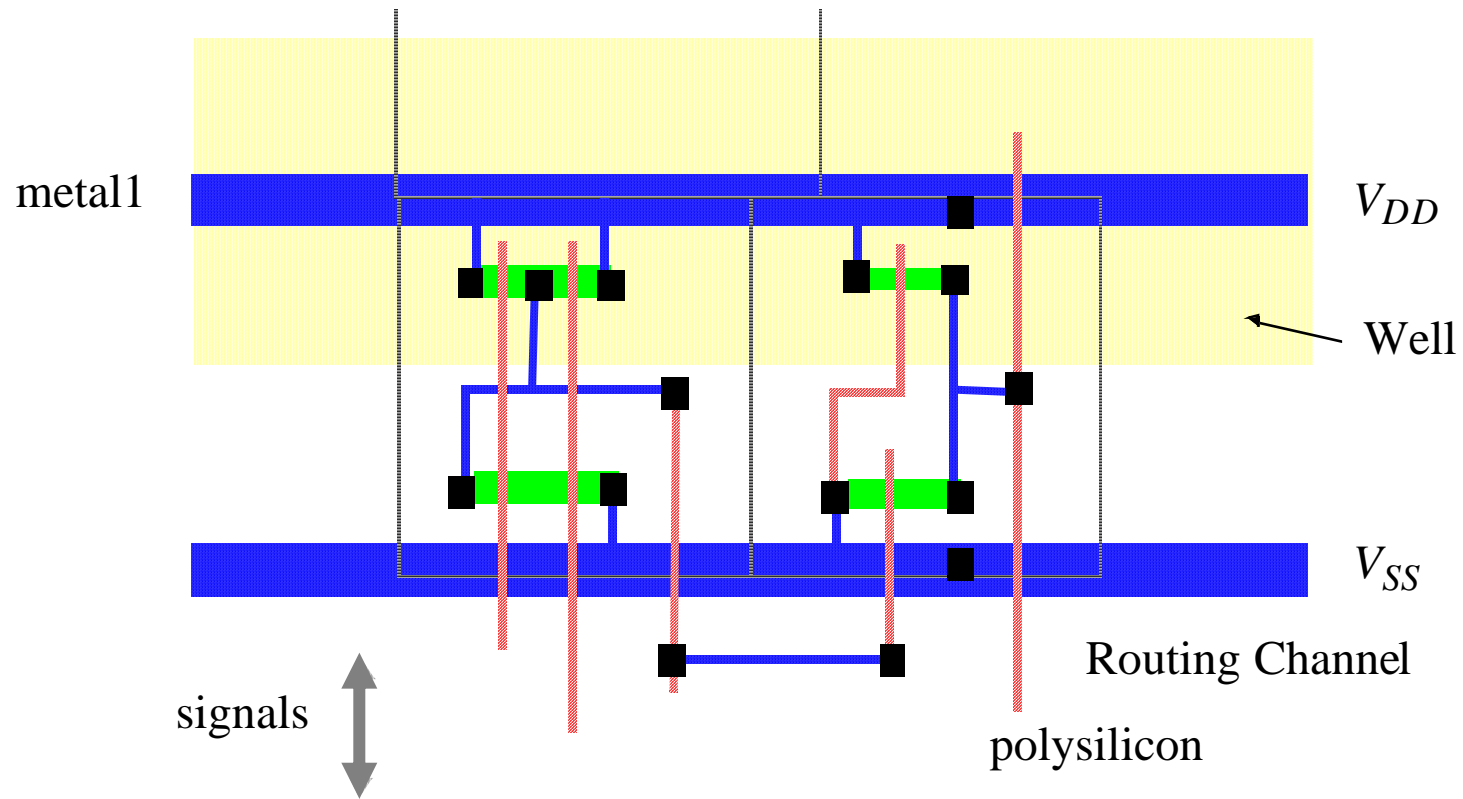
Full Adder Circuit



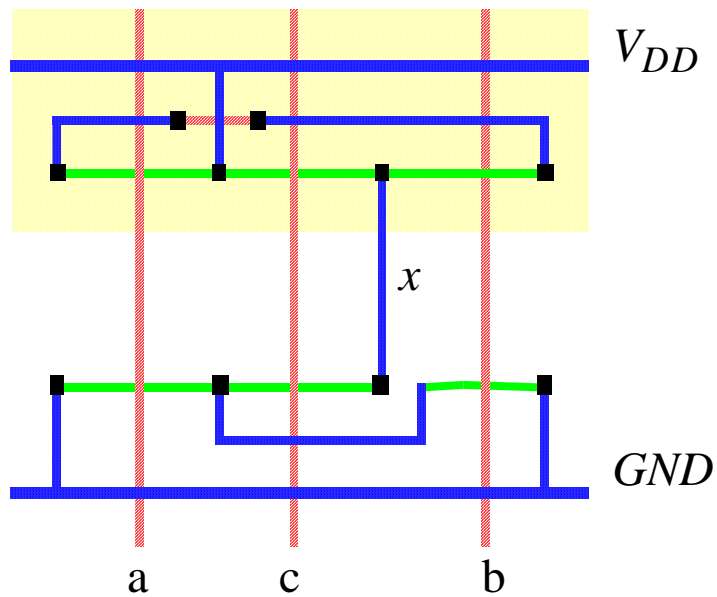
4-input NAND Gate



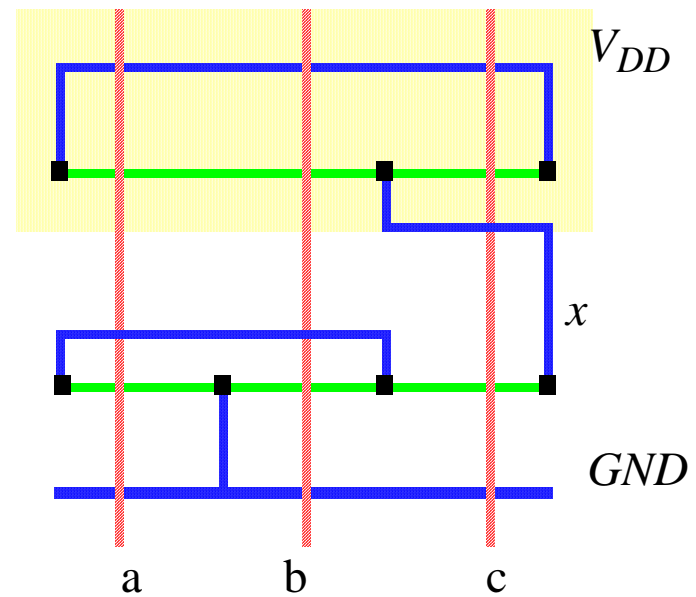
Standard Cell Layout Methodology



Two Versions of $(a+b).c$

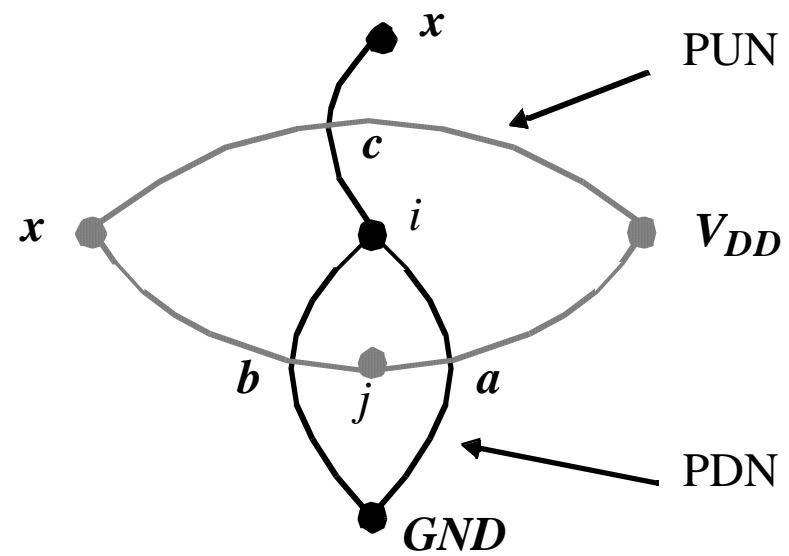
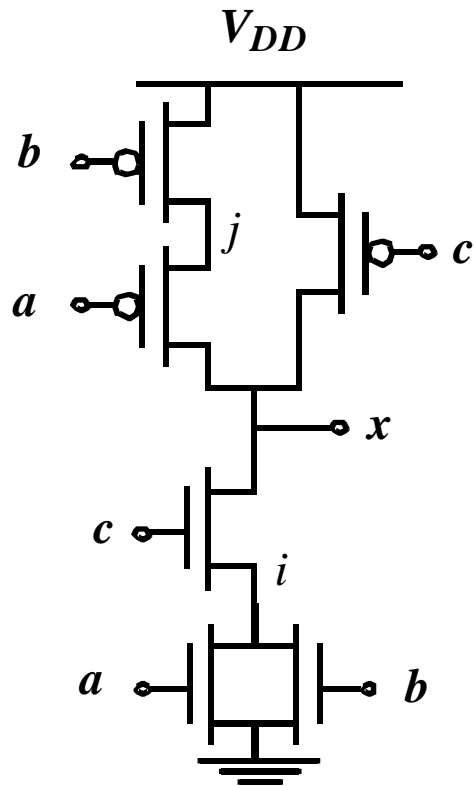


(a) Input order $\{a\ c\ b\}$

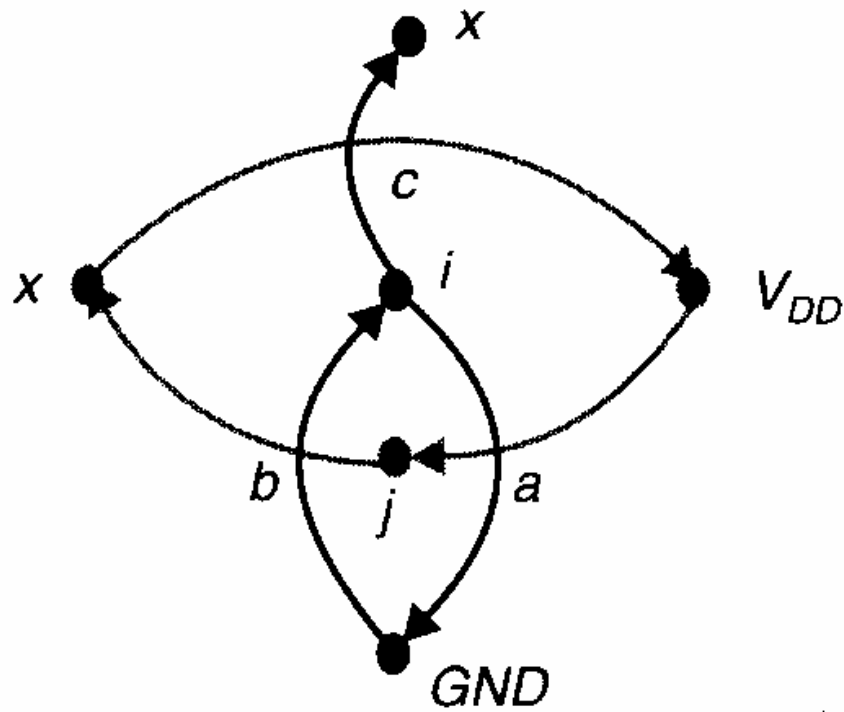


(b) Input order $\{a\ b\ c\}$

Logic Graph

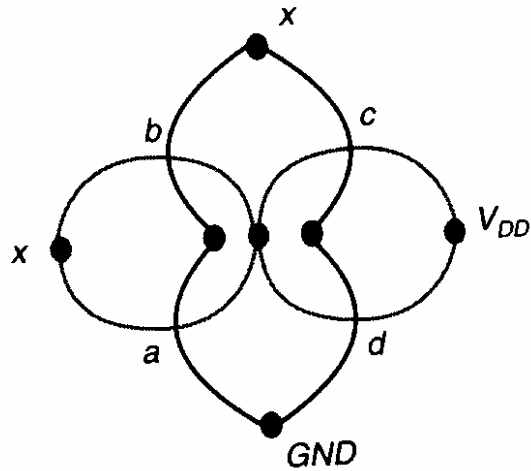


Consistent Euler Path

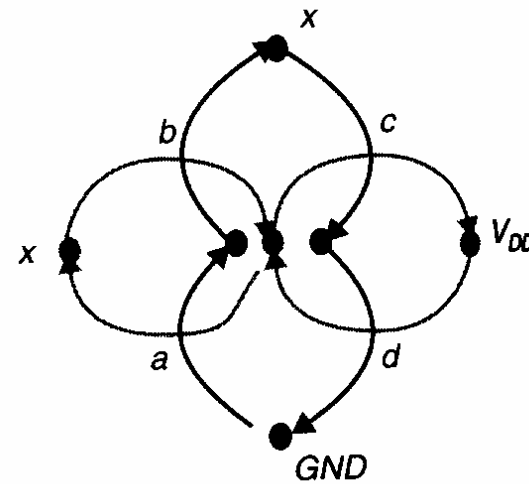


$\{a\ b\ c\}$

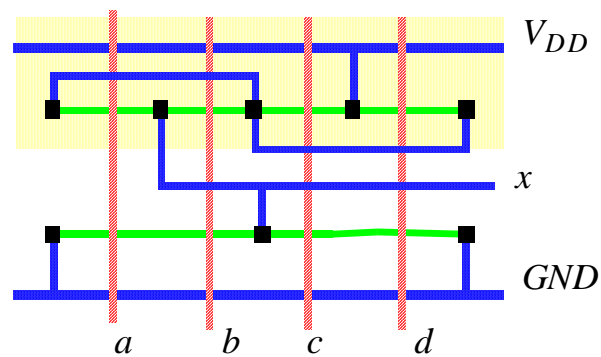
Example: $x = ab+cd$



(a) Logic graphs for $\overline{ab+cd}$



(b) Euler Paths $\{a b c d\}$

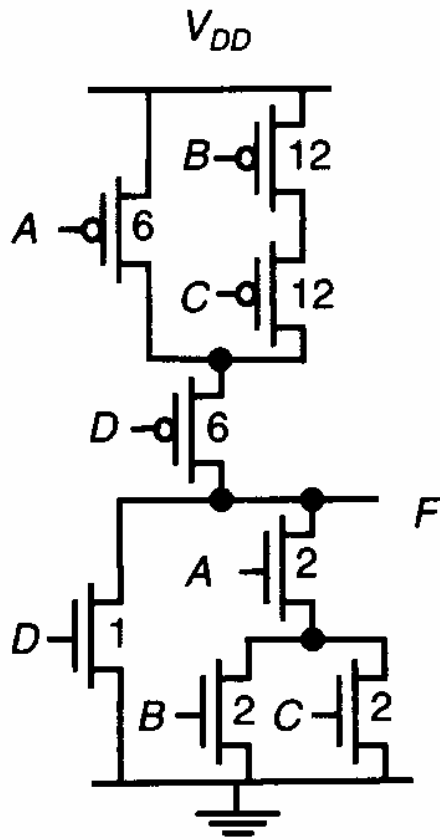


(c) stick diagram for ordering $\{a b c d\}$

Properties of Complementary CMOS Gates

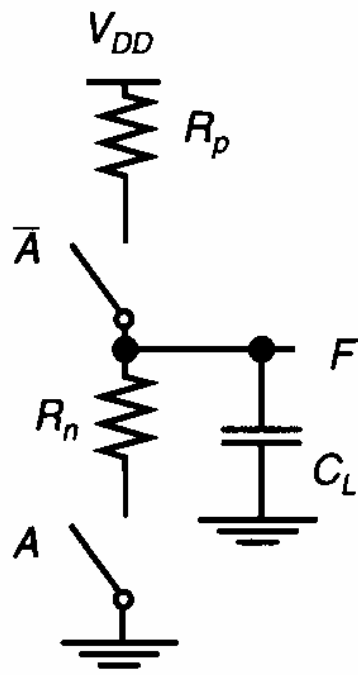
- High noise margin
 - V_{OH} and V_{OL} are at V_{DD} and G_{ND} , respectively
- No static power consumption
 - In steady state, no direct path between V_{DD} and V_{SS}
- Comparable rise and fall times under appropriate scaling of PMOS and NMOS transistors

Transistor Sizing

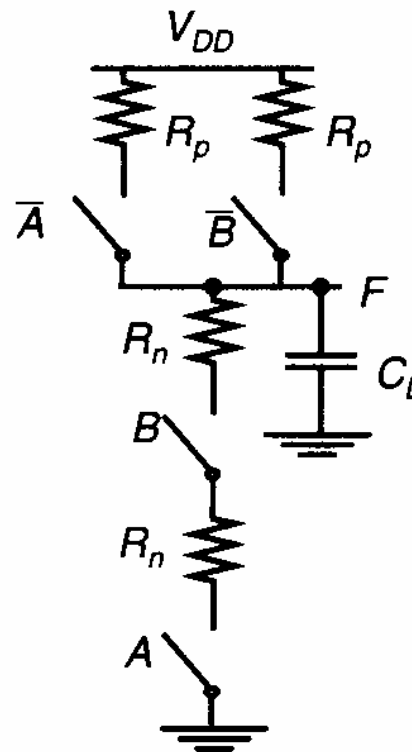


- For symmetrical response (dc, ac)
- For performance
- Input dependent
- Focus on worst-case

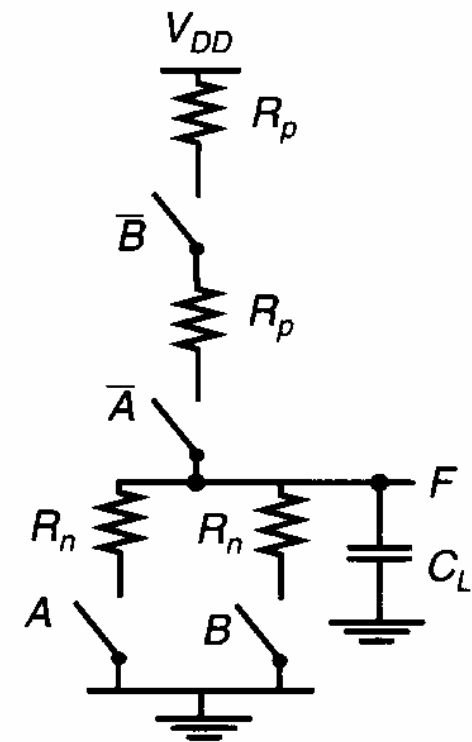
Propagation Delay Analysis - The Switch Model



(a) Inverter

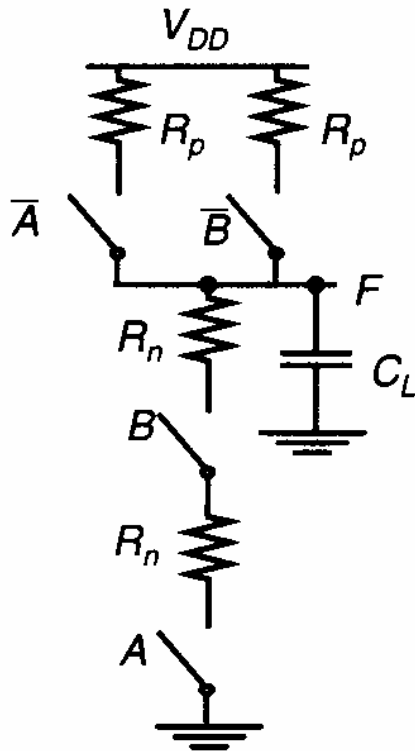


(b) Two-input NAND



(c) Two-input NOR

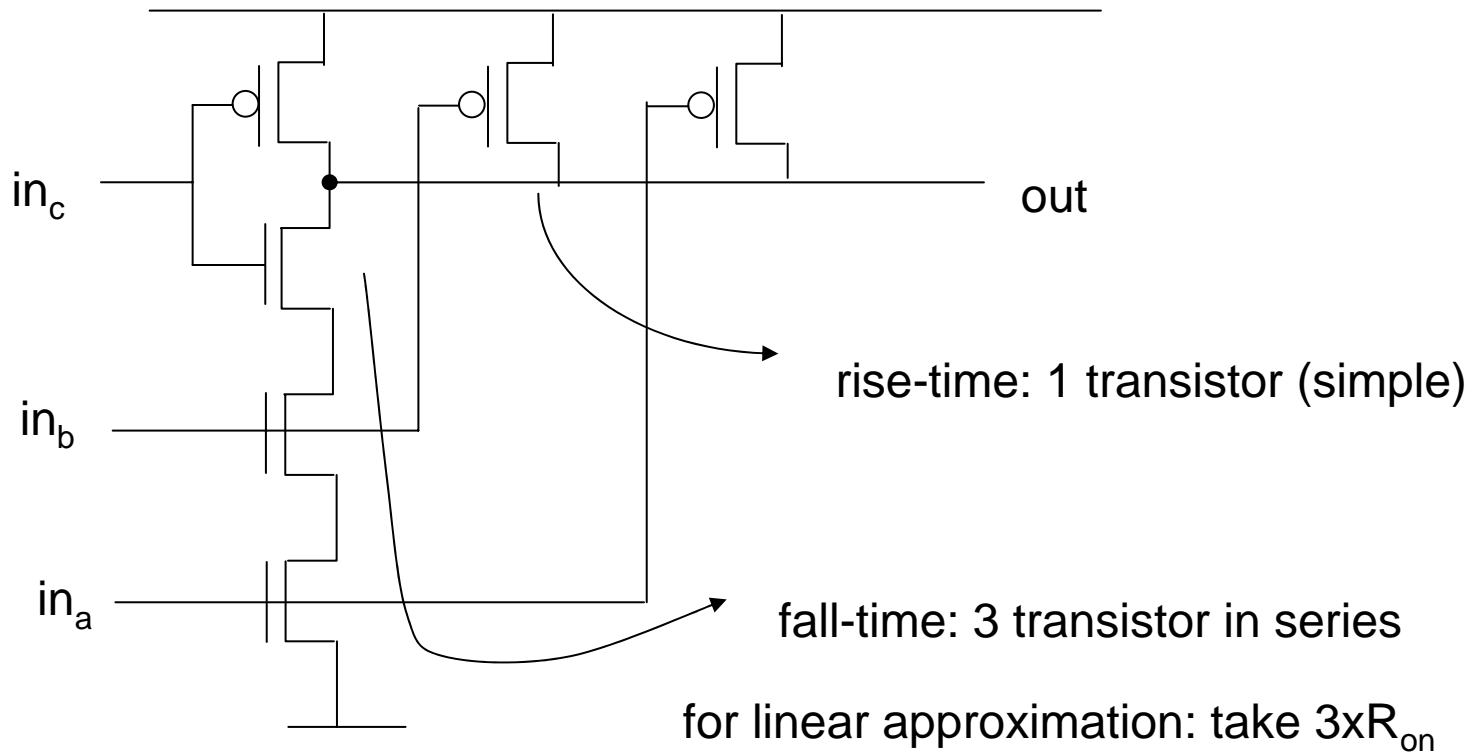
Analysis of Propagation Delay



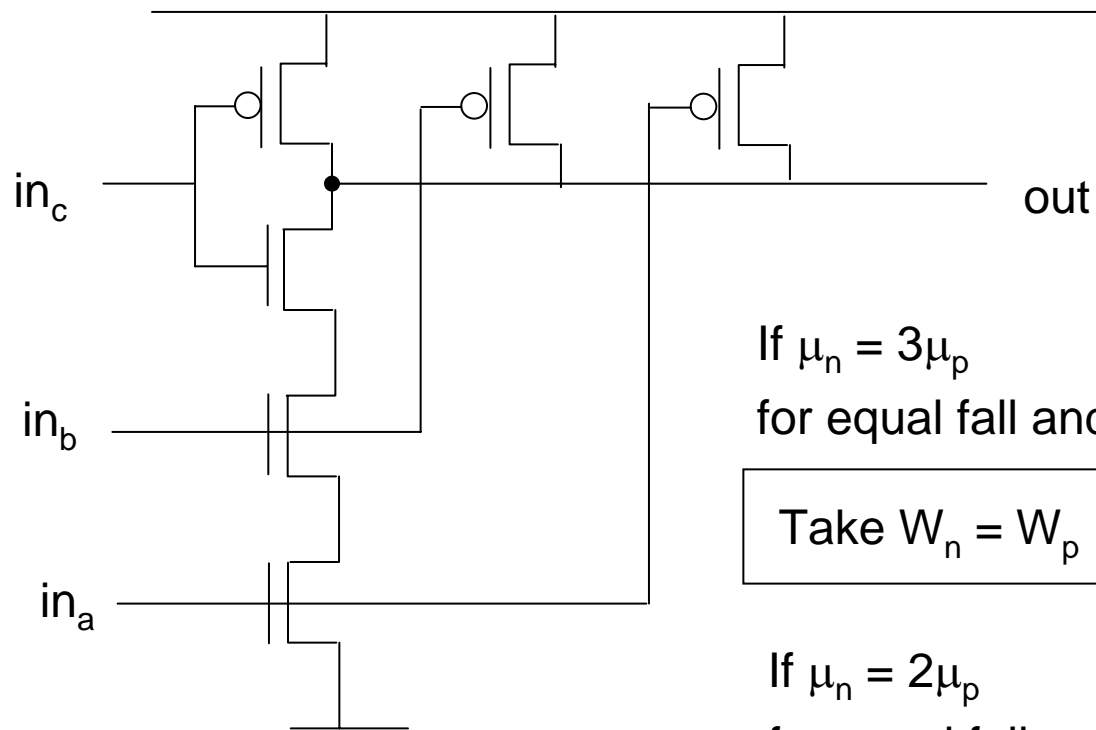
(b) Two-input NAND

- Assume C_L dominates
- Assume $R_n = R_p =$ resistance of minimum sized NMOS inverter
- For t_{pLH}
 - Worst case when only one PMOS pulls up the output node
 - $t_{pLH} \propto R_p C_L$
- For t_{pHL}
 - Worst case when two NMOS in series
 - $t_{pHL} \propto 2R_n C_L$

3-Input NAND Gate



3-Input NAND Gate



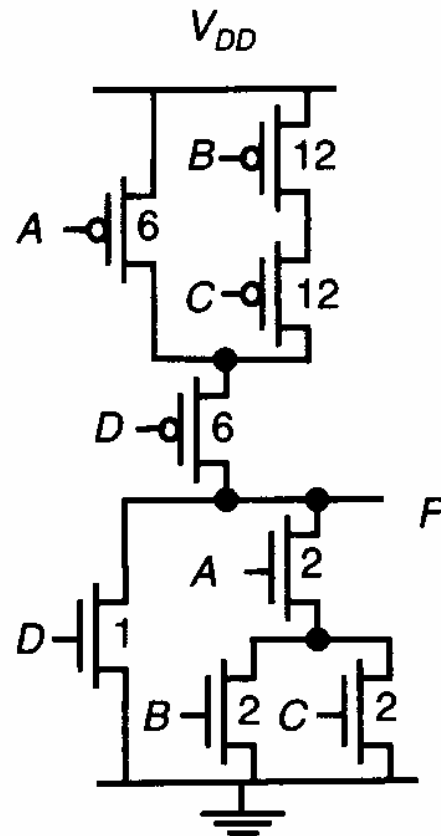
If $\mu_n = 3\mu_p$
for equal fall and rise time:

$$\text{Take } W_n = W_p$$

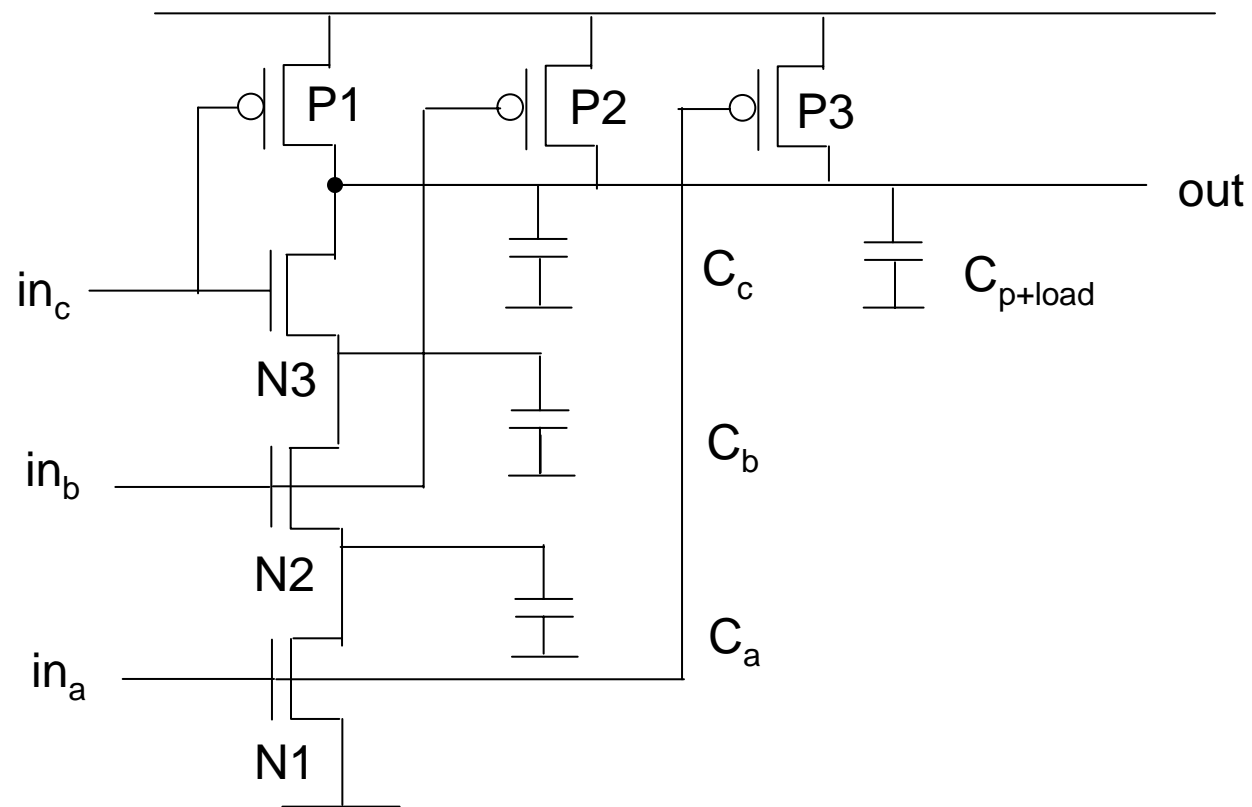
If $\mu_n = 2\mu_p$
for equal fall and rise time:

$$\text{Take } W_n = (3/2)W_p$$

Design for Worst Case



3-input NAND Gate with Parasitic Capacitors



Worst Case Approximation Using Lumped RC Model

(We ignore the constant term 0.69 or 1.22)

$$\begin{aligned}t_{df} &= \sum R_{pulldown} \times \sum C_{pulldown} \\ &= (R_{N1} + R_{N2} + R_{N3}) \times (C_a + C_b + (C_c + C_{p+load}))\end{aligned}$$

Penfield-Rubenstein Model (Elmore Delay Model)

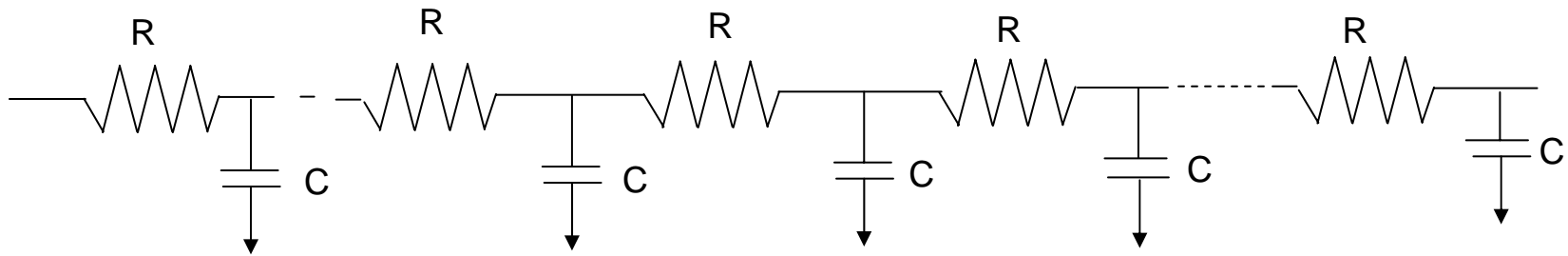
$$t_d = \sum R_i C_i$$

with: C_i = capacitance at node i

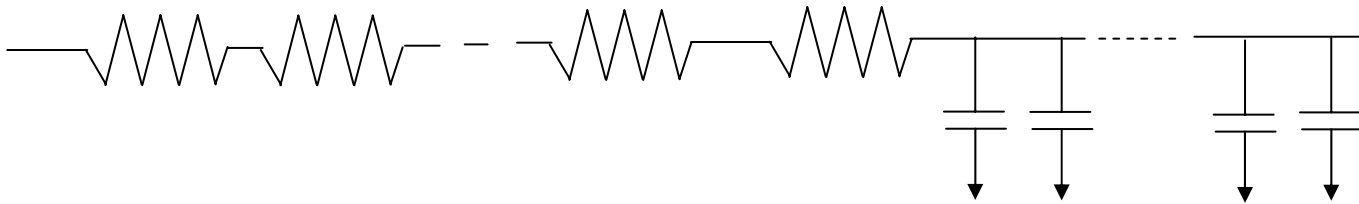
R_i = total resistance between C_i and supply

$$t_{df} = [R_{N1} C_a] + [(R_{N1} + R_{N2}) C_b] + \\ [(R_{N1} + R_{N2} + R_{N3})(C_c + C_{p+load})]$$

Distributed RC Effects



$$t_n = \frac{RC \times n(n+1)}{2} \approx \frac{nR \cdot nC}{2}$$



Worst case under lumped model: $t_n = nR \cdot nC$

Comparison

RP-Model

$$t_{df} = [R_{N1}C_a] + [(R_{N1}+R_{N2})C_b] + [(R_{N1}+R_{N2}+R_{N3})C_c] +$$

n transistors in series

$$\frac{[(R_{N1} + R_{N2} + R_{N3})C_{p+load}]}{}$$

$$R_N C n(n+1)/2 + [(R_{N1}+R_{N2}+R_{N3})C_{p+load}]$$

With $R_{N1} = R_{N2} = R_{N3} = R_N$
and $C_a = C_b = C_c = C$

Lumped-Model

$$R_N C n^2 + [(R_{N1}+R_{N2}+R_{N3})C_{p+load}]$$

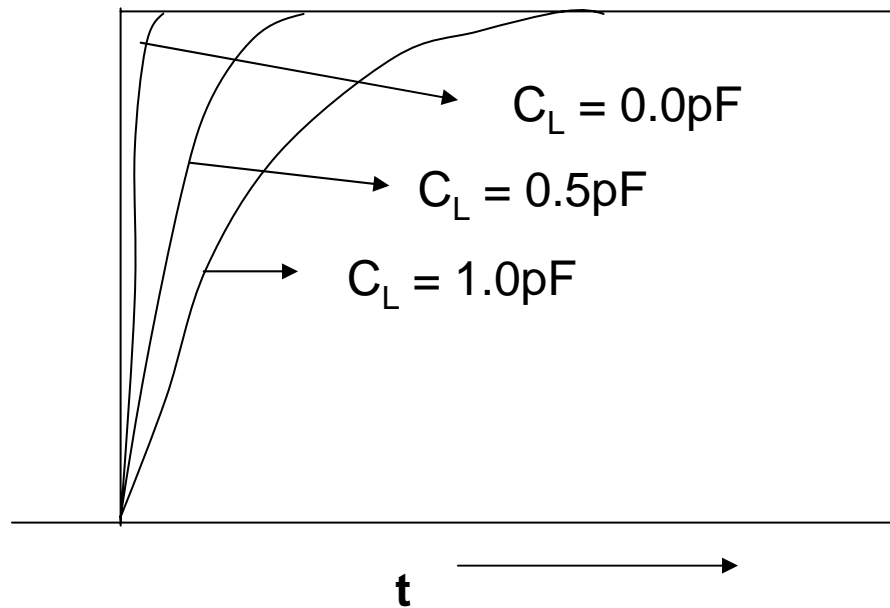
Macro Modeling

$$t_d = [R_{N1} C_a] + [(R_{N1} + R_{N2}) C_b] + [(R_{N1} + R_{N2} + R_{N3}) C_c] +$$
$$\frac{[(R_{N1} + R_{N2} + R_{N3}) C_p + [(R_{N1} + R_{N2} + R_{N3}) C_{load}]}{}$$

The diagram illustrates the mapping of terms in the delay equation to 'Internal delay' and 'External load'. A curved arrow points from the first three terms of the equation to the label 'Internal delay'. A straight arrow points from the first term of the fraction to 'Internal delay', and another straight arrow points from the second term of the fraction to 'External load'.

$$t_d = T_{d, \text{internal}} + \lambda \times C_{\text{load}}$$

Effect of Loading

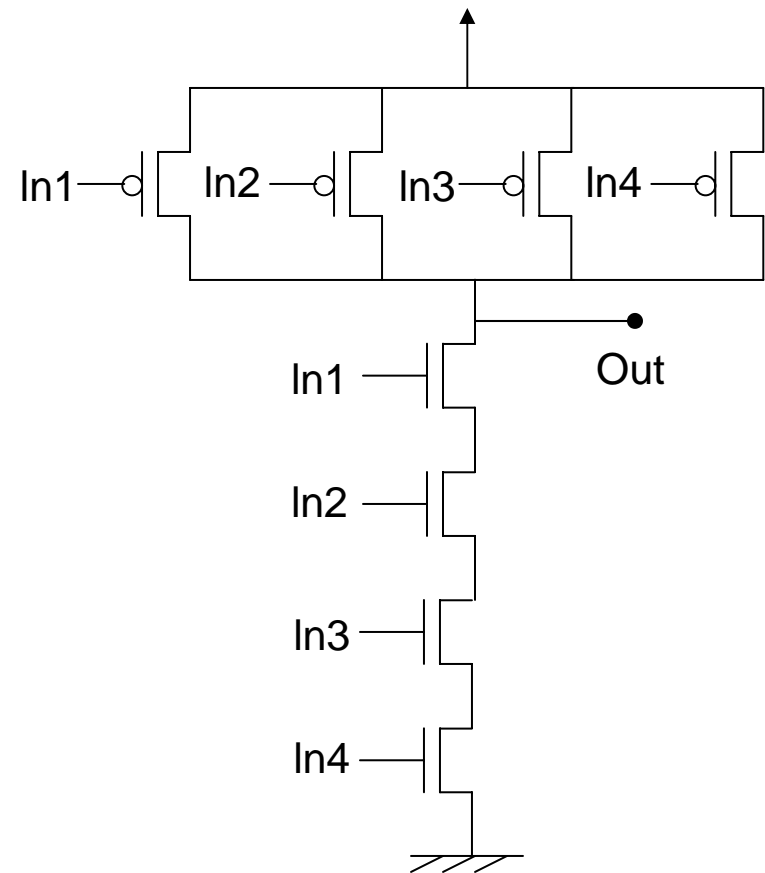


$$t_d = t_{d, \text{internal}} + \lambda \times C_{\text{load}}$$

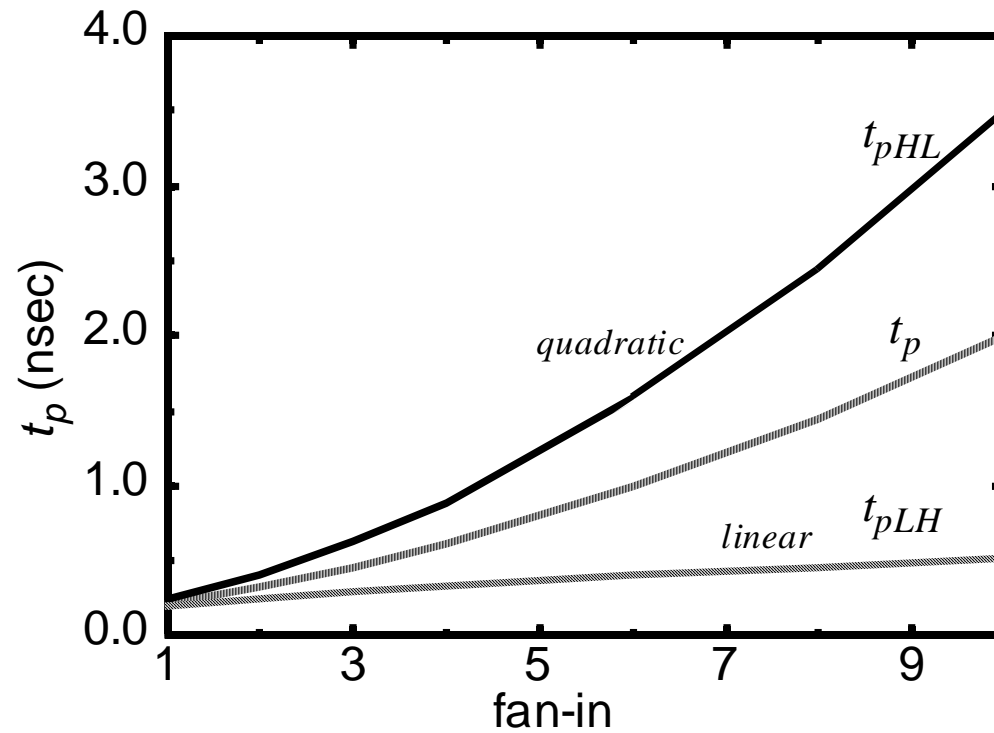
Effect of Fan-In and Fan-Out on Delay

$$t_d = a_1 FI + a_2 FI^2 + a_3 FO$$

- Fan-out: number of gates connected
 - 2 gate capacitance per fan-out
- Fan-in: number of inputs to a gate
 - Quadratic effect due to increasing resistance and capacitance



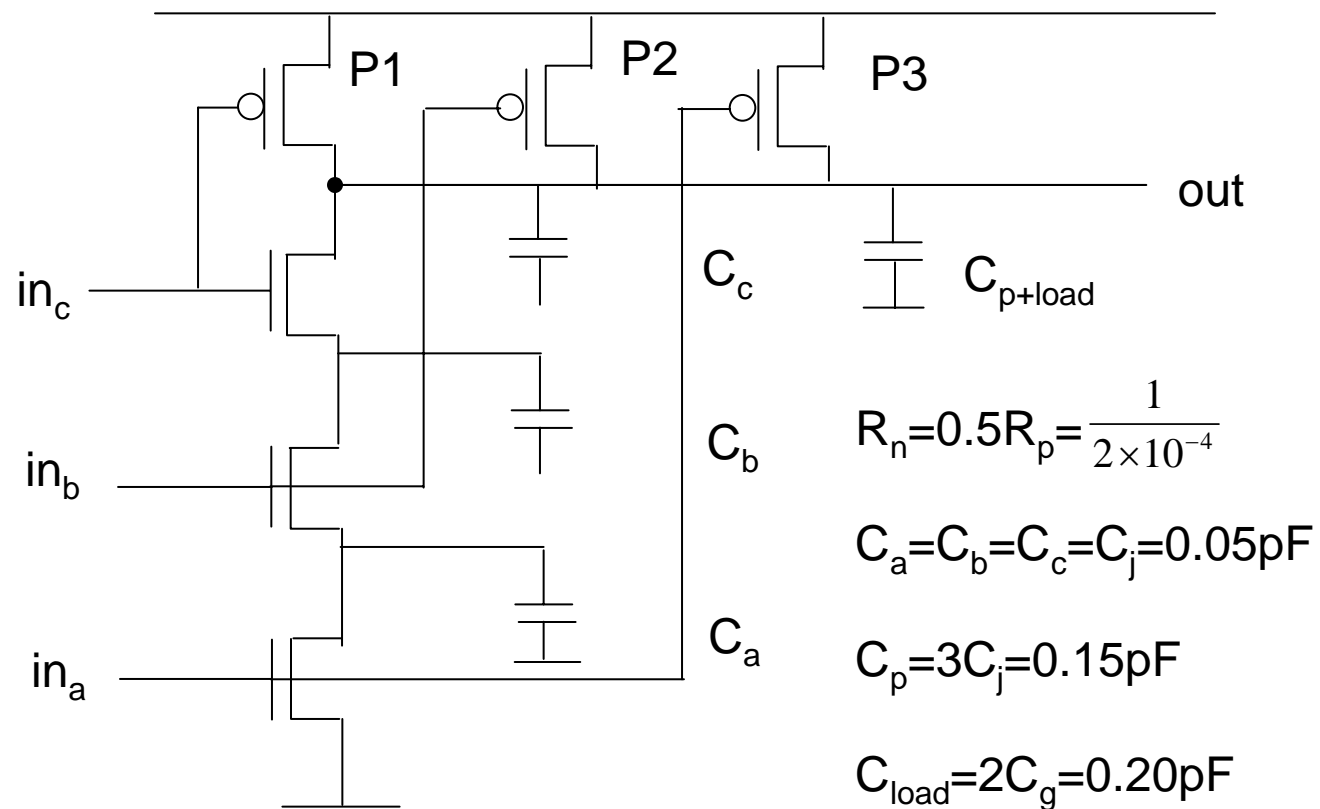
t_p as a function of Fan-In



AVOID LARGE FAN-IN GATES! (Typically not more than $FI < 4$)

Example

3-Input NAND gate with Parasitic Capacitors



Worst Case Approximation by Lumped Model

$$t_{dr} = R_p \times (C_c + C_{p+load}) = 10000 \times 0.4 \times 10^{-12} = 4.0\text{ns}$$

$$t_{df} = \Sigma R_{pulldown} \times \Sigma C_{pulldown}$$

$$= (R_{N1} + R_{N2} + R_{N3}) \times (C_a + C_b + (C_c + C_{p+load}))$$

$$= (3 \times 5000) \times (3 \times 0.05 + 0.15 + 0.20) \times 10^{-12}$$

$$= 7.5\text{ns}$$

Penfield-Rubenstein Model

$$t_{dr} = R_p \times (C_c + C_{p+load}) = 10000 \times 0.4 \times 10^{-12} = 4.0\text{ns}$$

$$\begin{aligned} t_{df} &= [R_{N1} C_a] + [(R_{N1} + R_{N2}) C_b] + [(R_{N1} + R_{N2} + R_{N3})(C_c + C_{p+load})] \\ &= 5000 \times 0.05\text{pF} + 10000 \times 0.05\text{pF} + 15000 \times 0.4\text{pF} = 6.75\text{ns} \end{aligned}$$

Worst Case Approximation by Lumped Model

Make $W_n = 2W_p$

$$t_{dr} = R_p \times (C_c + C_{p+load}) = 10000 \times 0.45 \times 10^{-12} = 4.5\text{ns}$$

$$t_{df} = \sum R_{pulldown} \times \sum C_{pulldown}$$

$$= (R_{N1} + R_{N2} + R_{N3}) \times (C_a + C_b + (C_c + C_{p+load}))$$

$$= (3 \times 2500) \times (3 \times 0.10 + 0.15 + 0.20) \times 10^{-12}$$

$$= 4.875\text{ns}$$

Penfield-Rubenstein Model

Make $W_n = 2W_p$

$$t_{dr} = R_p \times (C_c + C_{p+load}) = 10000 \times 0.45 \times 10^{-12} = 4.5\text{ns}$$

$$\begin{aligned} t_{df} &= [R_{N1} C_a] + [(R_{N1} + R_{N2}) C_b] + [(R_{N1} + R_{N2} + R_{N3})(C_c + C_{p+load})] \\ &= 2500 \times 0.10\text{pF} + 5000 \times 0.10\text{pF} + 7500 \times 0.45\text{pF} = 4.125\text{ns} \end{aligned}$$

Rewriting Penfield-Rubenstein Equation

$$t_d = [R_{N1} C_a] + [(R_{N1} + R_{N2}) C_b] + \\ [(R_{N1} + R_{N2} + R_{N3})(C_c + C_{p+load})]$$

⇒

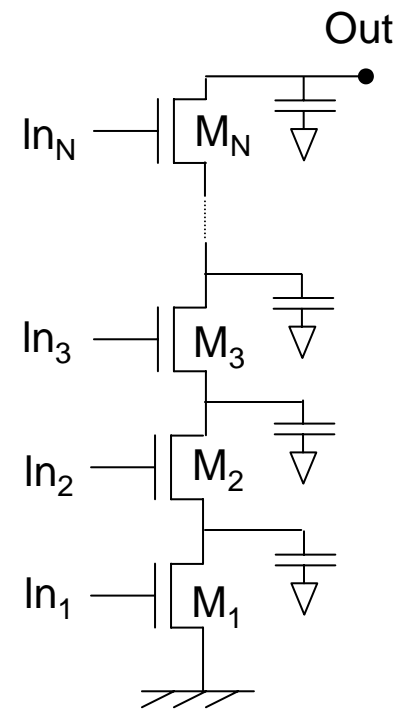
$$t_d = [R_{N1}(C_a + C_b + C_c + C_{p+load})] + \\ [R_{N2}(C_b + C_c + C_{p+load})] + \\ [R_{N3}(C_c + C_{p+load})]$$

$$t_d = \sum R_{ij} C_{\text{downstream-}i}$$

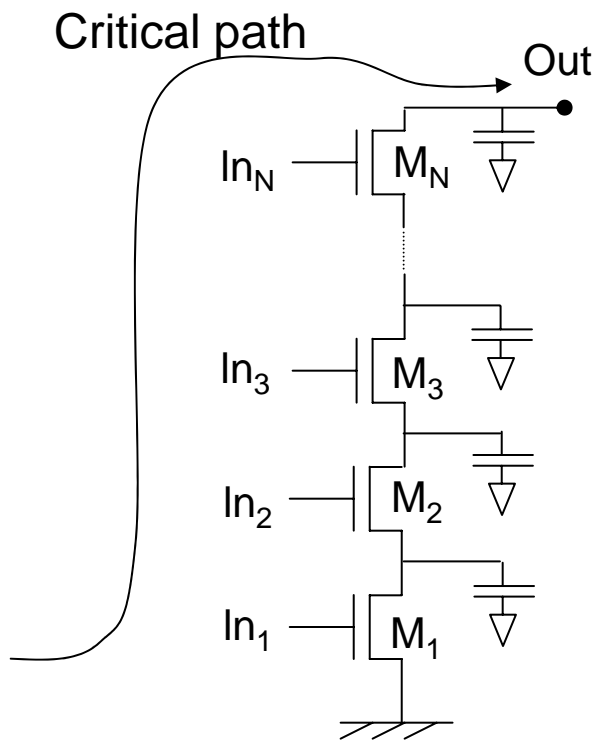
with: $C_{\text{downstream-}i}$ = downstream capacitance at node i
 R_{ij} = resistance at node i

Progressive Sizing

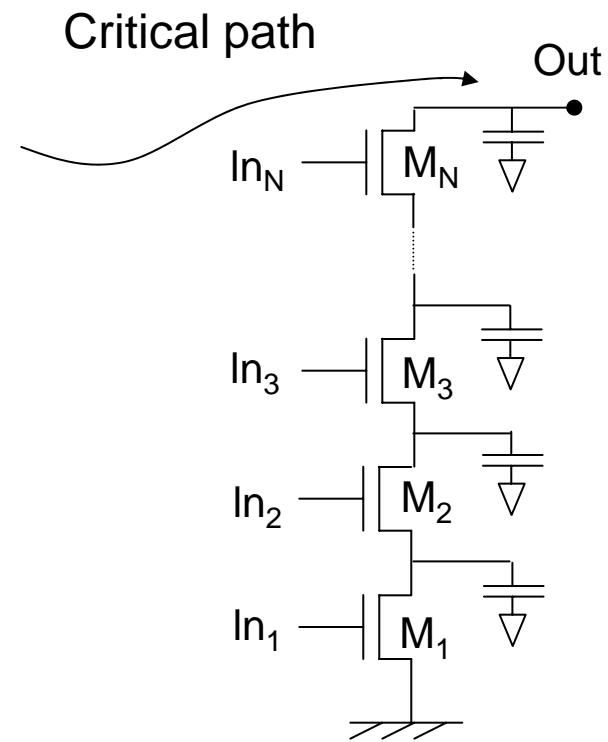
- When parasitic capacitance is significant (e.g., when fan-in is large), needs to consider distributed RC effect
- Increasing the size of M1 has the largest impact in terms of delay reduction
- $M_1 > M_2 > M_3 > \dots > M_N$



Delay Optimization by Transistor Ordering



Critical signal next to supply



Critical signal next to output