E-Core Inductor Design

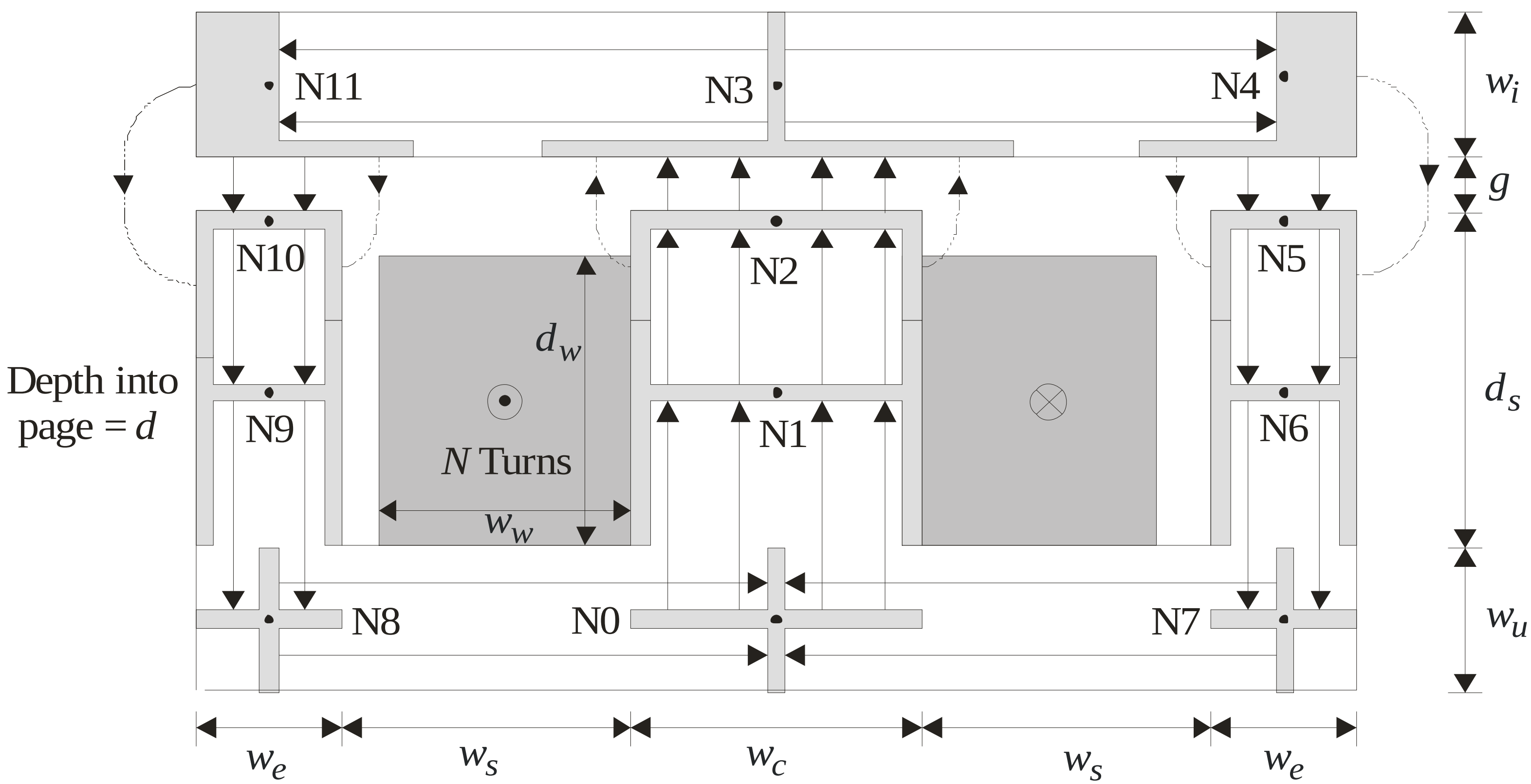
S.D. Sudhoff

Spring 2005

A Design Example

- Scenario. A power electronics converter design requires a filter inductor.
- Requirements
  - At maximum load, the average inductor current will be 3.0 A.
  - Current ripple less than 0.2 A.
  - The incremental inductance required is 5 mH.
  - The dc resistance of the inductor must be less than 0.1 Ω.
  - No dimension may exceed 15 cm.
  - It is desired to make the inductor as small as possible.
  - Packing factor not to exceed 0.7
  - Use 3C85 Ferrite

Review of EI Core Inductor



Design Choices

- Slot depth d_s
- Slot width w_s
- Winding depth d_w
- Winding width w_w
- Depth d
- Airgap g
- Turns N
- Wire Type (w_t)
- I-Core width w_i
- E-Core end width w_e
- E-Core center width w_u
- E-Core underside width w_u
- Magnetic Material (Not this Time)

- Total Variables: 12

Mathematical Formulation of Design Problem

- This is a constrained optimization problem.
- Minimize volume, subject to
  - Sufficient incremental inductance
  - Allowable packing factor
  - Appropriate dc resistance
  - Appropriate restriction on dimensions

Volume

- Volume may be readily expressed

$$h_o = w_u + d_s + g + w_i$$

$$w_o = 2w_e + 2w_s + w_c$$

$$d_o = d + 2w_w$$

$$v = h_0 w_0 d_0$$

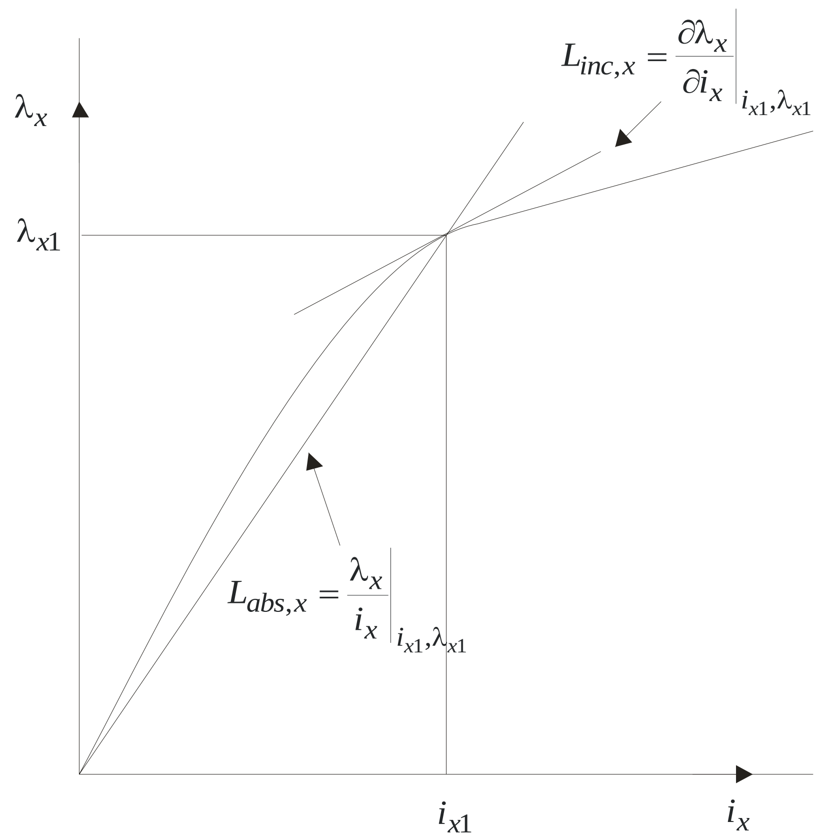
Formulation of Constraints: The Less Than Function

- To implement constraints, define the less than function

$$ltn(x, x_{\max}, \Delta x) = \begin{cases} 1 & x \le x_{\max} \\ \dfrac{1}{1 + \dfrac{x - x_{\max}}{|\Delta x|}} & x > x_{\max} \end{cases}$$

Constraint 1&2: Incremental inductance

- Recall



Constraint 1&2: Incremental Inductance

- Now

$$L_{inc} = \left.\frac{\partial \lambda}{\partial i}\right|_{o.p.} \approx \frac{\Delta \lambda}{\Delta i} = \frac{\lambda_{\max} - \lambda_{\min}}{i_{\max} - i_{\min}}$$

- To formulate as a constraint

$$\Delta\lambda_{\min} = L_{inc,required}(i_{\max} - i_{\min})$$

$$i_{\max} = 3.1$$
$$i_{\min} = 2.9$$

$$c_1 = analysis \;\; converges$$
$$c_2 = ltn(\lambda_{\max} - \lambda_{\min}, \Delta\lambda_{\min}, 0.01\Delta\lambda_{\min})$$

Constraint 3: Packing Factor

- Current density is normally a constraint, but this is okay by virtue of wire picked
- Packing factor is a measure of how well we fill the slot

$$pf = \frac{a_c N}{w_w d_w}$$

$$c_3 = ltn(pf, pf_{\max}, 0.01 pf_{\max})$$

- a_c is a function of the wire type

Aside: Wire Selection (Any of These Will Carry the Current)

Wire Type	**AWG**	**Area (μin^2)**
1	22	503
2	20	804
3	18	1276
4	16	2027
5	14	3227
6	12	5158
7	10	8155
8	8	12970
9	6	20610

Constraint 4: Resistance

- Recall DC resistance

$$r = \frac{N^2(2d + 2w_c + \pi w_w)}{\sigma_c p_f d_w w_w}$$

- We will formulate constraint 4 as

$$c_4 = ltn(r, r_{\max}, 0.01 r_{\max})$$

Constraint 5: Maximum Dimension

$$h_o = w_u + d_s + g + w_i$$

$$w_o = 2w_e + 2w_s + w_c$$

$$d_o = d + 2w_w$$

$$m_o = \max(h_o, w_o, d_o)$$

$$c_5 = ltn(m_o, m_{o\max}, 0.01 m_{o,\max})$$

Objective Function

- One approach

$$c = \min(c_1, c_2, c_3, c_4, c_5)$$

$$f = \begin{cases} c_1 + c_2 + c_3 + c_4 + c_5 - 5 & c < 1 \\ \dfrac{1}{\varepsilon + v} & c \geq 1 \end{cases}$$

Conclusion (Sort Of)

- We have thus transformed our design problem into the optimization problem

$$\text{maximize} \quad f(x)$$

$$x = \begin{bmatrix} g & d_s & w_s & f_{d_w} & f_{w_w} & w_i & f_{w_e} & f_{w_c} & f_{w_u} & d & N & w_t \end{bmatrix}$$

Conclusion (The Problem)

- A wide variety of optimization routines exist and are coded in standard math packages
- Examples: Matlab, Mathcadd, etc.
- The catch: you need a good initial guess