

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad \underline{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\underline{w} = a \underline{x} + b \underline{y} + c \underline{z} \quad \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

But if  $\underline{x}$ ,  $\underline{y}$ , and  $\underline{z}$  are orthogonal

$$\underline{x}^T \underline{w} = a \underbrace{\underline{x}^T \underline{x}}_0 + b \underbrace{\underline{x}^T \underline{y}}_0 + c \underbrace{\underline{x}^T \underline{z}}_0$$

$$a = \frac{\underline{x}^T \underline{w}}{\underline{x}^T \underline{x}} \quad \text{and if } \underline{x}^T \underline{x} = 1 \Rightarrow a = \underline{x}^T \underline{w} \quad b = \underline{y}^T \underline{w} \quad c = \underline{z}^T \underline{w}$$

$$\underline{y}^T \underline{y} = 1$$

$$\text{and } \underline{z}^T \underline{z} = 1$$

$a, b, c$  are like the Fourier Series coefficients  
 $\underline{x}, \underline{y}, \underline{z}$  are like the sine waves at different frequencies

$$\underline{x}^T \underline{w} = \sum_{k=1}^3 w_k x_k \quad \underline{y}^T \underline{w} = \sum_{k=1}^3 w_k y_k$$

$$\underline{z}^T \underline{w} = \sum_{k=1}^3 w_k z_k$$

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \underbrace{e^{-j2\pi \frac{k}{T} t}}_{S_k^*(t)} dt$$

where:  $T = \int_{-T/2}^{T/2} |S_k(t)|^2 dt$   
 $S_k(t) = e^{j2\pi \frac{k}{T} t}$