Supplementary Notes on Time-Invariance

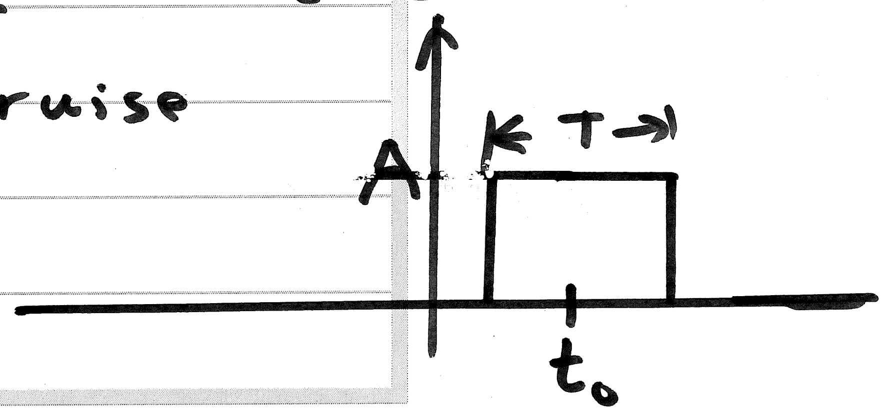
$$\mathrm{rect}(t) = \begin{cases} 1, & -0.5 < t < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{rect}\left(\frac{t-t_0}{T}\right) = 1 \qquad -\frac{1}{2} < \frac{t-t_0}{T} < \frac{1}{2}$$

multiply "both" sides by T: $-\frac{T}{2} < t - t_0 < \frac{T}{2}$

add t_0 to "both" sides: $t_0 - \frac{T}{2} < t < t_0 + \frac{T}{2}$

$$A\,\mathrm{rect}\left(\frac{t-t_0}{T}\right) = \begin{cases} A, & t_0 - \frac{T}{2} < t < t_0 + \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$



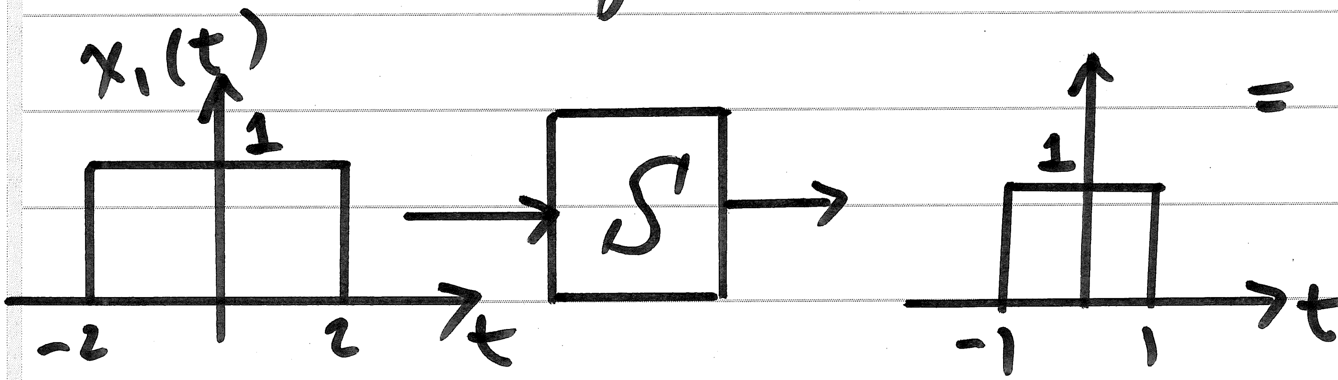
Example 1.16 on Text pg. 52

<u>System</u> : $y(t) = x(2t)$

- Is system Time-Invariant (TI)?
- Already proved $y(t) = x(at)$ is <u>not</u> TI for any value of a ($\neq 1$)
- Nonetheless, consider Example 1.16
- Consider input: $x_1(t) = \text{rect}\left(\frac{t}{4}\right)$

Output is: $y_1(t) = x_1(2t) = \text{rect}\left(\frac{2t}{4}\right)$

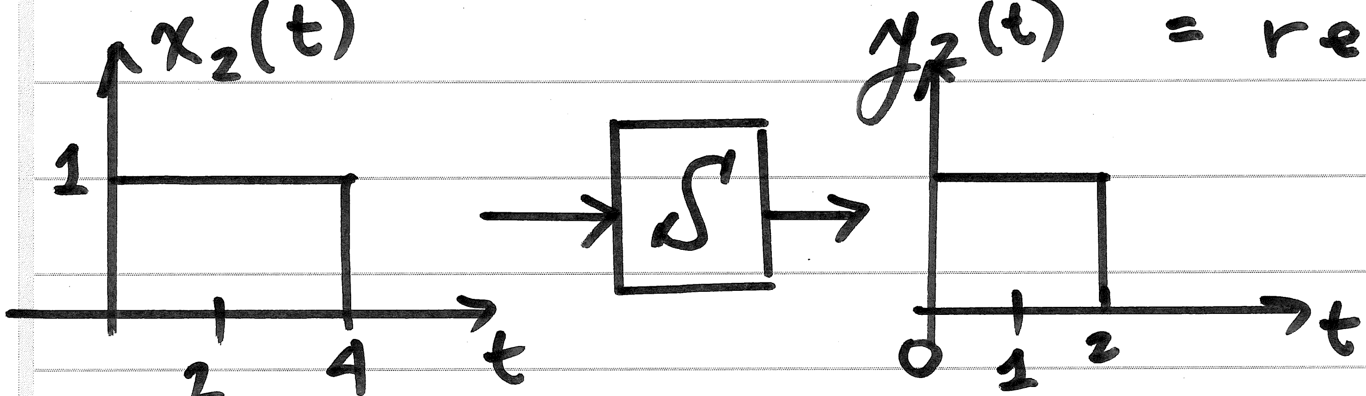
$= \text{rect}\left(\frac{t}{2}\right)$



• <u>**Now**</u>: consider input $x_2(t) = x_1(t-2)$

$$x_2(t) = x_1(t-2) = \text{rect}\left(\frac{t-2}{4}\right)$$

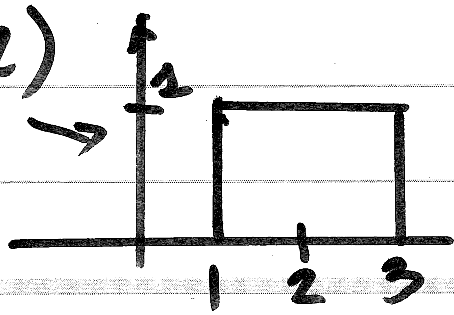
• <u>Output</u> is: $y_2(t) = x_2(2t) = \text{rect}\left(\frac{2t-2}{4}\right)$



$$= \text{rect}\left(\frac{t-1}{2}\right)$$

• <u>But</u> since $x_2(t) = x_1(t-2)$, if the system was TI, the output should have been

$y_1(t-2)$



THUS: System <u>not</u> TI