System Examples

1. $y(t) = x(at)$

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = x(at)$$

Linear? Yes

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow \boxed{S} \longrightarrow y(t) = a_1 x_1(at) + a_2 x_2(at)$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

where:

$$x_i(t) \longrightarrow \boxed{S} \longrightarrow y_i(t) = x_i(at)$$

$i = 1, 2$

TI? No, not time-invariant

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = x(at)$$

$$x(t-t_0) \longrightarrow \boxed{S} \longrightarrow z(t) = x(at-t_0)$$

Is $z(t) \stackrel{?}{=} y(t-t_0)$? No

$$x(at-t_0) \neq x\big(a(t-t_0)\big) = x(at-at_0)$$

See Example 1.16 in text on pg. 52

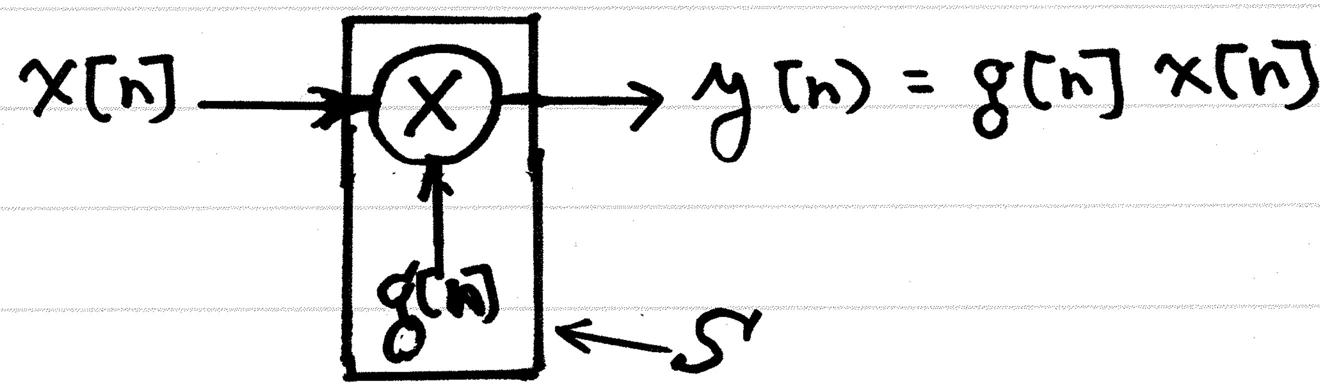
System is stable.

System is not causal if $a < 0$

Same for DT system $y[n] = x[an]$, a, integer

Example 2. DT System

$$y[n] = g[n]\,x[n]$$



Linear? Yes, due to distributive property of multiplication

$$a_1 x_1[n] + a_2 x_2[n] \longrightarrow \boxed{S} \longrightarrow y[n] =$$

$$g[n]\{a_1 x_1[n] + a_2 x_2[n]\}$$

$$= a_1 g[n] x_1[n] + a_2 g[n] x_2[n]$$

$$= a_1 y_1[n] + a_2 y_2[n]$$

TI? No, system is not Time-Invariant

$$x[n-n_0] \longrightarrow \boxed{S} \longrightarrow z[n] = g[n]\, x[n-n_0]$$

Is $z[n] \stackrel{?}{=} y[n-n_0]$? $\boxed{y[n] = g[n]\, x[n]}$

$$g[n]\, x[n-n_0] \neq g[n-n_0]\, x[n-n_0]$$

System is not TI

System is BIBO Stable if $|g[n]| < \infty \ \forall n$

System is causal (it is memoryless)

<u>Examples:</u> $g[n] = \cos(\omega_0 n)$

$g[n] = n$ See Hmwk. 2

Comments:

- Similar observations hold with respect to the CT System $y(t) = g(t)\, x(t)$
  - See Eq. 1.17 pg. 54 where $g(t) = t$
- Even though this system is not both Linear and Time-Invariant (LTI), it is still a very useful system in practice
- The only ramification of not being LTI is that the output is not related to the input through convolution with the impulse response of the system = but still useful

Example 3. Square-law System

$$x(t) \longrightarrow \boxed{S} \longrightarrow y(t) = x^2(t)$$

Linear? No. Superposition does not apply.

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow \boxed{S} \longrightarrow y(t) \Rightarrow$$

$$\Rightarrow \{a_1 x_1(t) + a_2 x_2(t)\}^2 =$$

$$= \underbrace{a_1^2}_{\text{violates linearity}} x_1^2(t) + \underbrace{a_1 a_2 x_1(t) x_2(t) 2}_{\text{violates linearity}} + \underbrace{a_2^2}_{\text{violates linearity}} x_2^2(t)$$

$$\neq a_1 x_1^2(t) + a_2 x_2^2(t)$$

- See Example 1.18 in text on pg. 54
- System is TI: $x(t-t_0) \longrightarrow \boxed{S} \longrightarrow x^2(t-t_0)$
- stable and causal too
- Similar observations hold wrt DT system

$$y[n] = x^2[n]$$

- Again, even though not LTI, this system is used often in practice, as we will see later in this course
- See handout posted at course website for further examples of systems

Example: Hard-Limiter

$$y(t) = \begin{cases} V_{max}, & \text{if } x(t) > V_{max} \\ x(t), & \text{if } -V_{max} < x(t) < V_{max} \\ -V_{max}, & \text{if } x(t) < -V_{max} \end{cases}$$

- used to remove noisy spikes in radios (FM)
- used, after amplitude-scaling to fit signal into range of quantizer in process of A/D conversion

- System is TI but not linear

- You can show nonlinearity with a single counter-example $\Rightarrow$ $x_1(t) = A_1 u(t)$ and $x_2(t) = A_2 u(t)$, where $A_1 < V_{max}$ and $A_2 < V_{max}$ BUT $A_1 + A_2 > V_{max}$

$$x_1(t) \longrightarrow \boxed{S} \longrightarrow y_1(t) = A_1\, u(t)$$

$$x_2(t) \longrightarrow \boxed{S} \longrightarrow y_2(t) = A_2\, u(t)$$

$$x_1(t) + x_2(t) \longrightarrow \boxed{S} \longrightarrow y(t) = V_{max}\, u(t)$$

$$\neq y_1(t) + y_2(t)$$

$$= (A_1 + A_2)\, u(t)$$

Concrete Example:
A_1 = 3
A_2 = 4
V_max = 5

Example. Integrator

$$y(t) = \int_{t-T_b}^{t+T_a} x(\tau)\, d\tau = \begin{cases} \text{area under} \\ \quad x(t) \\ \text{from } \; t-T_b \\ \text{to } \; t+T_a \end{cases}$$

$T_a > 0$, $T_b > 0$. note: $T_b = \infty$ is valid

System is LTI.

Linearity:

$$y(t) = \int_{t-T_b}^{t+T_a} \left(a_1 x_1(\tau) + a_2 x_2(\tau)\right) d\tau$$

$$= a_1 \int_{t-T_b}^{t+T_a} x_1(\tau)\, d\tau + a_2 \int_{t-T_b}^{t+T_a} x_2(\tau)\, d\tau$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

TI: input $x(t-t_0)$:

$$z(t) = \int_{t-T_b}^{t+T_a} x(\tau - t_0)\, d\tau$$

• change of variables: $\lambda = \tau - t_0$

$d\lambda = d\tau \qquad \tau = \lambda + t_0$

new limits:

$$\lambda \Big]_{t-T_b-t_0}^{t+T_a-t_0}$$

THUS:

$$z(t) = \int_{t-t_0-T_b}^{t-t_0+T_a} x(\lambda)\, d\lambda = y(t-t_0)$$

System is LTI