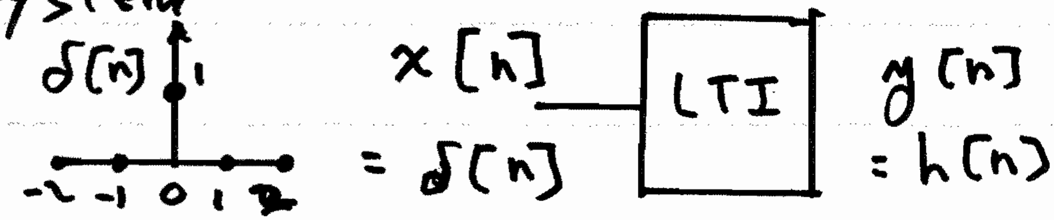
- Additional Features of Impulse Response of LTI System



- BIBO Stability ⇒ necessary condition:

If: $$\sum_{n=-\infty}^{\infty} |h[n]| < \infty,$$ then system is BIBO stable

<u>Proof:</u> $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Bounded input: $|x[n]| < A_{max_{in}} < \infty \quad \forall n$

Triangle Inequality: $|a+b| \le |a| + |b|$

<u>Thus:</u> $$|y[n]| < \sum_k |h[k]\, x[n-k]| = \sum_k |h[k]|\, |x[n-k]|$$

- Since $|x[n-k]| < A_{max_{in}}$ $\forall n$ $\forall k$, it follows

$$|y[n]| < \left\{ \sum_{k=-\infty}^{\infty} |h[k]| \right\} A_{max_{in}}$$

Thus, if $B = \sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then $|y[n]| < \infty$

Note: IF $\sum_{k=-\infty}^{\infty} |h[k]| = \infty$ and you select input

as $x[n] = \dfrac{h^*[-n]}{|h[-n]|}$ $\therefore$ $y[n] = \sum_{k=-\infty}^{\infty} h[k] \dfrac{h^*[-(n-k)]}{|h[-(n-k)]|}$

$\boxed{|x[n]|=1 \ \forall n}$

Consider $y[0]$:

$$y[0] = \sum_{k=-\infty}^{\infty} \frac{h[k]\, h^*[k]}{|h[k]|} = \sum_{k=-\infty}^{\infty} \frac{|h[k]|^2}{|h[k]|} = \sum_{k=-\infty}^{\infty} |h[k]| = \infty$$

So, it is a <u>necessary</u> condition for stability

- Causality: $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] =$

$\ldots + h[-2]\, x[n+2] + h[-1]\, x[n+1] + h[0]\, x[n] +$

$+ h[1]\, x[n-1] + h[2]\, x[n-2] + \ldots$

- Thus, if $h[n] = 0$ for $n < 0$, then system is causal $\Rightarrow$ does <u>not</u> depend on future inputs

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- For CT LTI Systems:
- <u>BIBO Stability</u>: $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$
- <u>Causality</u>: $h(t) = 0$ for $t < 0$