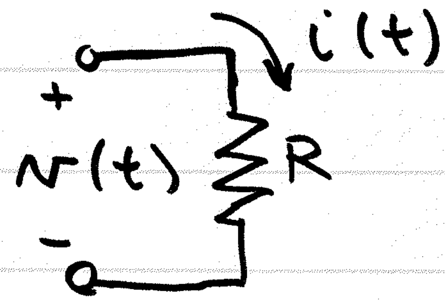
Signal Basics

Signal: $x(t)$

Energy: $E_x = \int_{-\infty}^{\infty} x^2(t)\,dt$

Recall:



Energy dissipated in resistor:

$$\int_{-\infty}^{\infty} \frac{v^2(t)}{R}\,dt = \int_{-\infty}^{\infty} i^2(t)\,R\,dt$$

Can kind of view our definition of energy as energy per unit resistance

Transformations of Time-Variable

Time-Shift: $x(t-t_0)$

Time-Scaling: $x(at)$
$a>0$

$a>1$: compression
$a<1$: expansion

Time-Reversal: $x(-t)$

Combination of the above:

$$x(at+b) \quad \text{or} \quad x(\alpha(t-\beta))$$

- steps must be done in proper order

$\underline{x(at+b)}$: 1. shift to left by b

2. if $a<0$, flip about $t=0$ (time-reverse)

3. compress by a factor of $|a|$
(if $|a|<1$, actually an expansion)

$\underline{x(\alpha(t-\beta))}$: 1. if $\alpha<0$, flip about $t=0$

2. if $|\alpha|>1$, compress by a factor of α
if $|\alpha|<1$, expand by $1/|\alpha|$

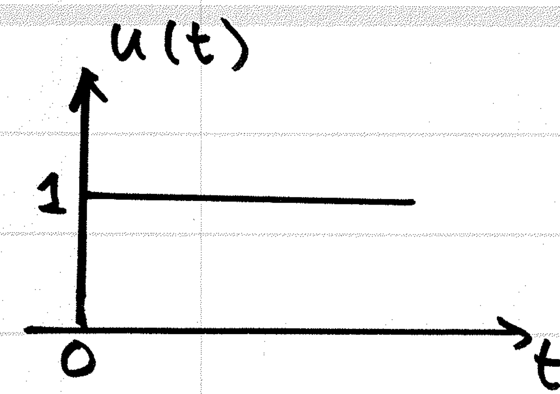
3. shift to right by β

See textbook examples and homework problems

See also matlab examples

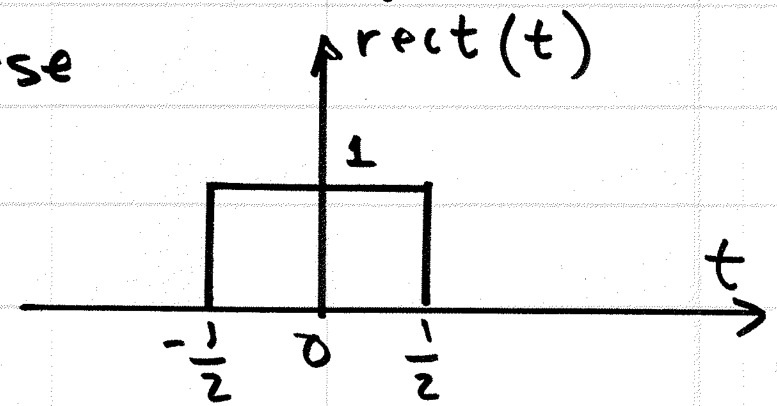
Basic signals

unit step: $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$



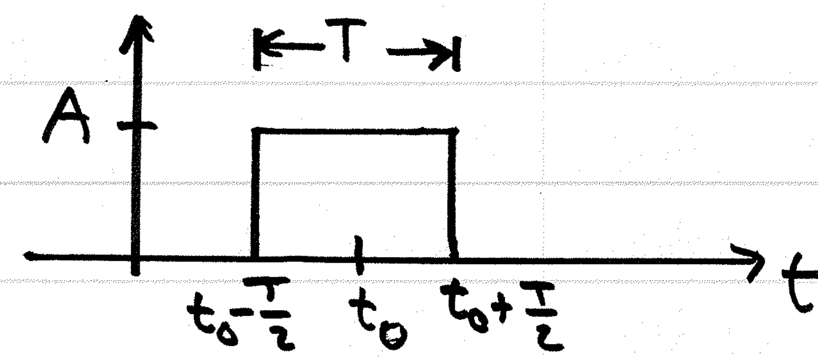
rectangle function: not defined in text, but we will use throughout course

$$\mathrm{rect}(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & |t| > 1/2 \end{cases}$$



$$\mathrm{rect}(t) = u(t + \tfrac{1}{2}) - u(t - \tfrac{1}{2})$$

$A\,\mathrm{rect}\left(\frac{t - t_0}{T}\right)$:



Supplementary Notes on Time-Invariance

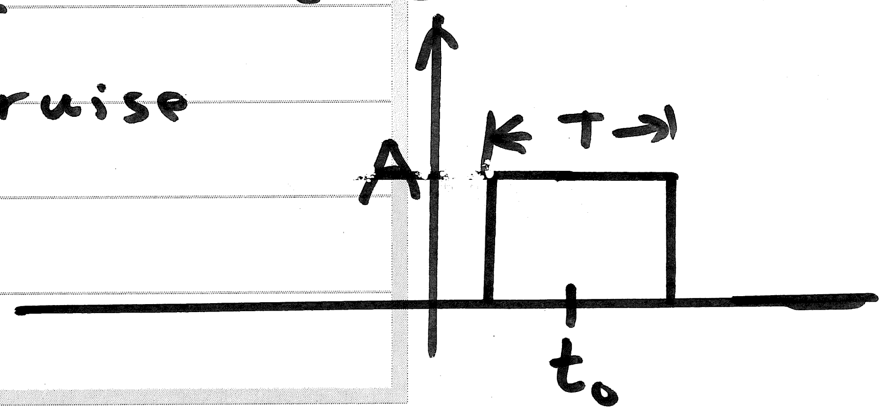
$$\text{rect}(t) = \begin{cases} 1, & -0.5 < t < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{rect}\left(\frac{t-t_0}{T}\right) = 1 \qquad -\frac{1}{2} < \frac{t-t_0}{T} < \frac{1}{2}$$

multiply "both" sides by T: $\quad -\frac{T}{2} < t - t_0 < \frac{T}{2}$

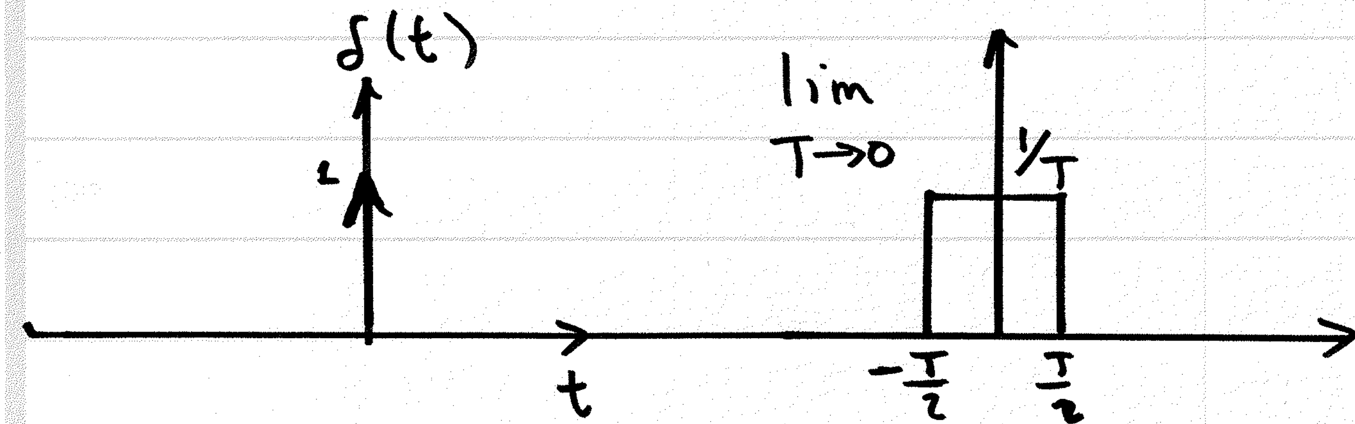
add t_0 to "both" sides: $\quad t_0 - \frac{T}{2} < t < t_0 + \frac{T}{2}$

$$A\,\text{rect}\left(\frac{t-t_0}{T}\right) = \begin{cases} A, & t_0 - \frac{T}{2} < t < t_0 + \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$



Dirac Delta Function

$$\delta(t) = \lim_{T \to 0} \frac{1}{T} \operatorname{rect}\left(\frac{t}{T}\right)$$



Properties of $\delta(t)$:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 = \int_{0^-}^{0^+} \delta(t)\,dt$$

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$ sifting property

$$\delta(t) = \frac{d\,u(t)}{dt}$$ see Text book

Exponential Signals

$$x(t) = e^{-at}\,u(t)$$

- a : can be real-valued, complex-valued, or purely imaginary => last case corresponds to sinewave and considered on next page
- consider a real-valued and positive
- Compute energy of $x(t)$:

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} e^{-2at}\,dt = \frac{-1}{2a}\, e^{-2at}\Big]_0^{\infty}$$

$$= \frac{1}{2a}\Big(\underbrace{e^{-\infty}}_{=0} - e^{0}\Big) = +\frac{1}{2a}$$

Other properties of $\delta(t)$ will be proved later

Sinewaves $\quad x(t) = e^{j\omega_0 t}$ = cos(ω_0t) + j sin(ω_0t)

$\omega_0 = 2\pi f_0$, f_0 is frequency in Hz (cycles per sec.)

period: $e^{j\omega_0(t+T)} = e^{j\omega_0 t} = e^{j\omega_0 t} e^{j2\pi}$

$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

$$\Rightarrow f_0 = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f_0}$$

see textbook for plots

By Euler's Formula, we have:

$$\cos(\omega_0 t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$\sin(\omega_0 t) = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

The nice thing about complex sinewaves is that the phase factors out as part of a complex amplitude:

$$x(t) = A e^{j(\omega_0 t + \theta)} = A e^{j\theta} e^{j\omega_0 t}$$

Of course, this is not true for a real-valued sinewave:

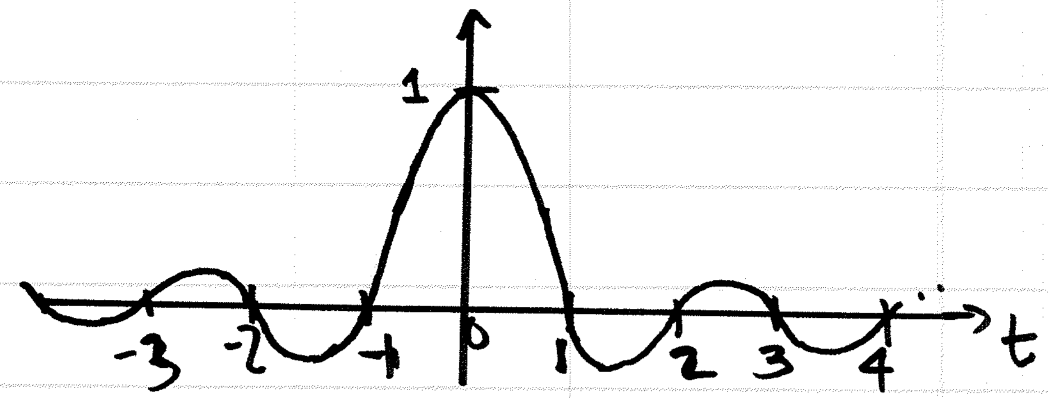
$$\cos(\omega_0 t + \theta) \neq \cos(\omega_0 t) \cos(\theta)$$

$$= \cos(\theta) \cos(\omega_0 t) - \sin\theta \sin(\omega_0 t)$$

Hence, why you solved circuits the way you did in ECE 201

<u>Sinc function</u> : very important function in many different fields of engineering (physics)

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



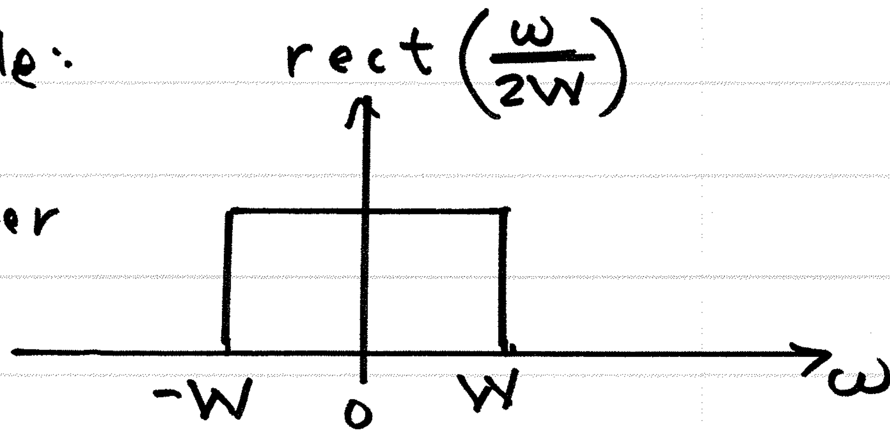
Can use L'Hospital's Rule to find value at $t=0$

OR: approximate $\sin(x)$ for $x \ll 1$ with first term in Taylor series expansion: $\sin(x)$.

$$\sin(x) \approx x \quad \text{for } x \ll 1$$

Thus: $$\lim_{t \to 0} \frac{\sin(\pi t)}{\pi t} = \frac{\pi t}{\pi t} = 1$$

- For each of the previously defined functions the independent variable could be something other than time t.
- e.g., could be position x $\Rightarrow$ $\text{sinc}(x)$
- later they will also be used as functions of frequency ω
- a very different interpretation when the independent variable is ω
- for example:



lowpass filter frequency response

Even and Odd Signals

- even: $x(-t) = x(t)$
- example:

$$x(t) = \cos(\omega_0 t)$$

symmetric about $t=0$

- odd-signal: $x(-t) = -x(t)$
- example:

$$x(t) = \sin(\omega_0 t)$$

anti-symmetric about $t=0$

- can decompose arbitrary signal into even and odd parts: $x(t) = x_e(t) + x_o(t)$

$$x_e(t) = \mathcal{E}v\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$$

$$x_o(t) = \mathcal{O}d\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$