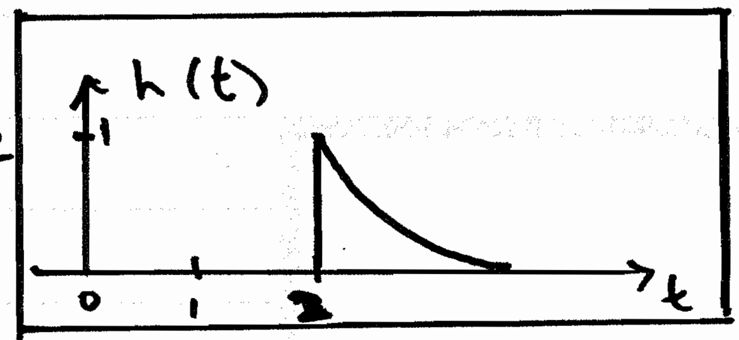
Prob. 2.40 $\quad y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau-2)\, d\tau$

$h(t) = ?$ $\quad$ let: $x(t) = \delta(t) \Rightarrow x(\tau-2) = \delta(\tau-2)$

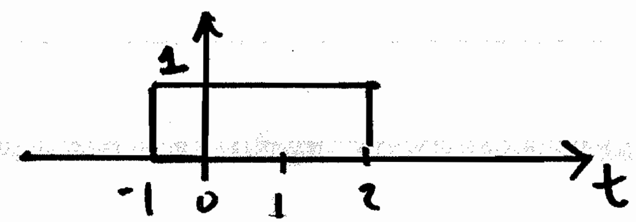
$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau = \int_{-\infty}^{t} e^{-(t-2)} \delta(\tau-2)\, d\tau$$

$$= e^{-(t-2)} \underbrace{\int_{-\infty}^{t} \delta(\tau-2)\, d\tau}_{=1 \text{ , if } t>2}$$

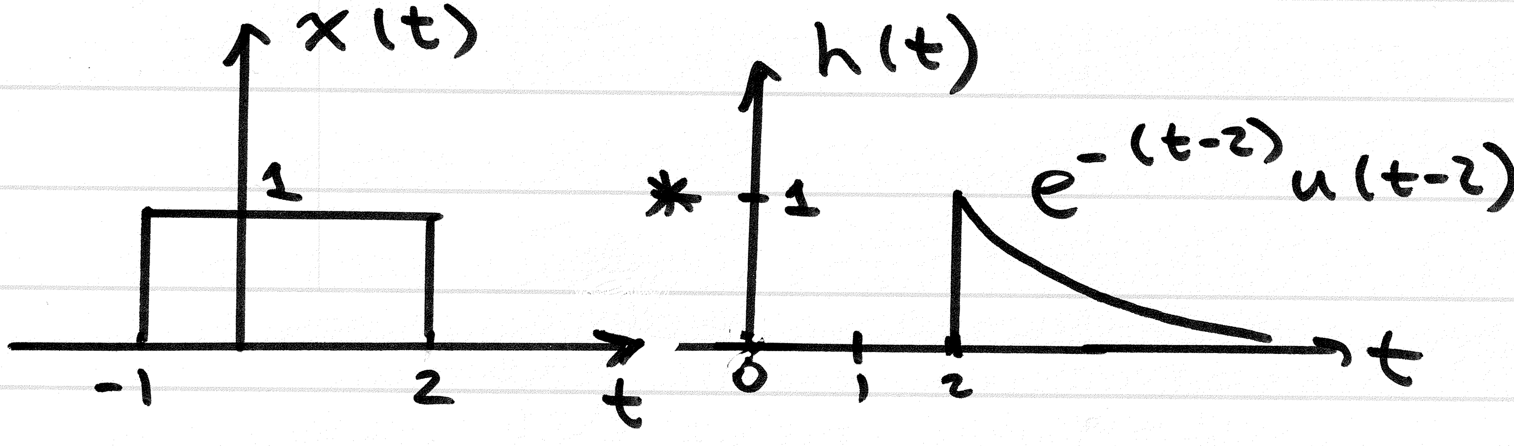


$$h(t) = e^{-(t-2)} u(t-2)$$

Find: $y(t)$ when $x(t) = u(t+1) - u(t-2) = \text{rect}\left(\frac{t-1/2}{3}\right)$

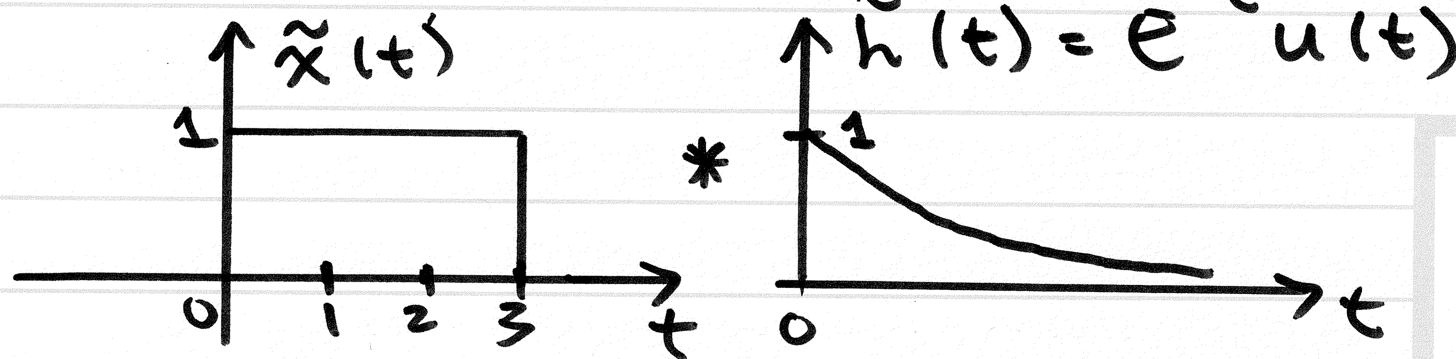


Prob. 2.40 (b)



$$y(t) = x(t) * h(t) = ?$$

Instead, convolve:



Note:

$$\tilde{x}(t) = u(t) - u(t-3)$$

Find $\tilde{y}(t) = \tilde{x}(t) * \tilde{h}(t)$ using known convolution results. Then, observe:

$$y(t) = \tilde{x}(t+1) * \tilde{h}(t-2)$$

$$= \tilde{y}(t - (\underbrace{-1}_{t_1=-1} + \underbrace{2}_{t_2=2}))$$

$$= \tilde{y}(t-1)$$

- where, again, $\tilde{y}(t)$ is the convolution result when both things being convolved start at $t=0$
- Now, basic convolution result from Prob. 2.22(a)

$$e^{-\alpha t}u(t) * e^{-\beta t}u(t) = \frac{1}{\beta-\alpha}\{e^{-\alpha t} - e^{-\beta t}\}u(t)$$

Text

- With $\beta=0$, we have result from Example 2.6

$$\left(e^{-\alpha t}u(t)\right) * u(t) = \frac{1}{\alpha}\left(1-e^{-\alpha t}\right)u(t)$$

- Now back to problem:

$$\tilde{y}(t) = \tilde{x}(t) * \tilde{h}(t) = \tilde{h}(t) * \tilde{x}(t)$$

$$= e^{-t}u(t) * \{u(t) - u(t-3)\}$$

$a=1$

$$= e^{-t}u(t) * u(t) - e^{-t}u(t) * u(t-3)$$

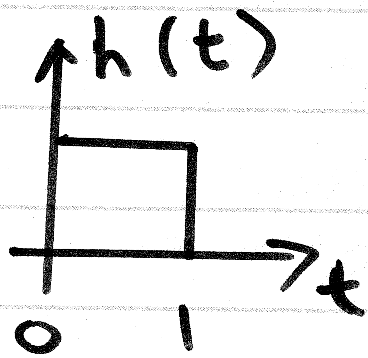
$a=1$

$$= \frac{1}{1}\left(1-e^{-at}\right)u(t) - \frac{1}{1}\left(1-e^{-a(t-3)}\right)u(t-3)$$

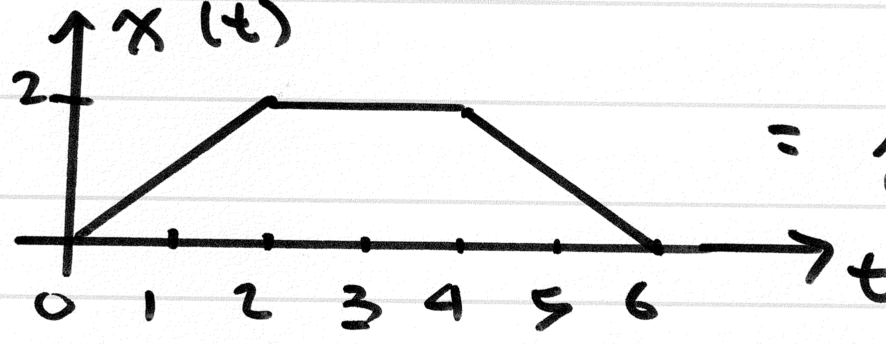
- Then, finally $y(t) = \tilde{y}(t-1)$:

$$y(t) = \left(1-e^{-(t-1)}\right)u(t-1) - \left(1-e^{-(t-4)}\right)u(t-4)$$

On Exam, might ask:



$*$



$= y(t) = ?$

$h(t) *$