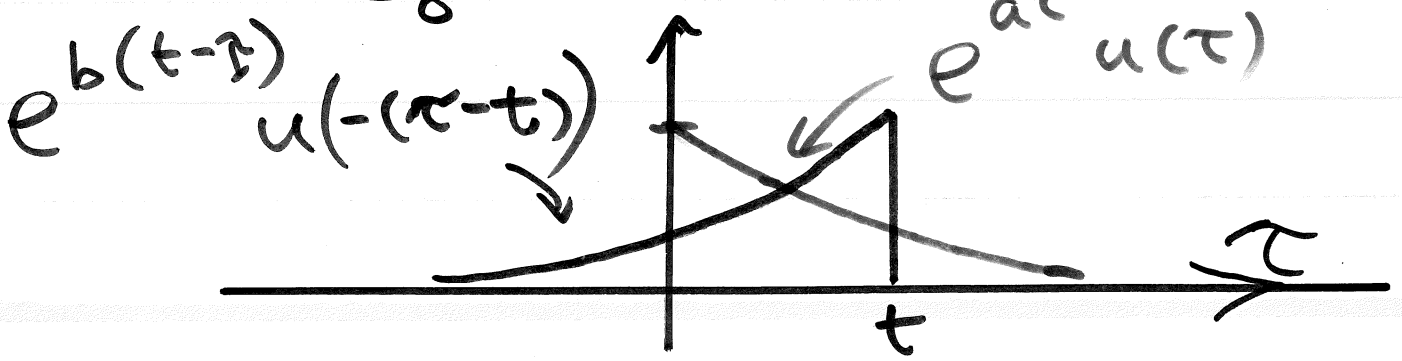
Prob. 2.22 (a) $(a=-\alpha,\ b=-\beta)$

$$e^{at}u(t) * e^{bt}u(t) = ? \quad \text{for } b \neq a$$

$$y(t)=\int_{-\infty}^{\infty} e^{a\tau}u(\tau)\, e^{b(t-\tau)}u(t-\tau)\,d\tau$$

$$= e^{bt}\int_{-\infty}^{\infty} e^{(a-b)\tau}u(\tau)\,u(-(\tau-t))\,d\tau$$

$$= e^{bt}\int_{0}^{t} e^{(a-b)\tau}\,d\tau \quad u(t)$$



$$y(t) = e^{bt}\left\{\frac{1}{a-b}e^{(a-b)\tau}\Big]_0^t\right\}$$

$$= \frac{e^{bt}}{a-b}\left\{e^{(a-b)t} - e^{0}\right\}$$

$$= \left\{\frac{e^{at} - e^{bt}}{a-b}\right\}u(t) \quad = y(t)$$

$$= e^{at}u(t) * e^{bt}u(t) \qquad a \neq b$$

$$= g(t) \quad \text{(notation)} = y(t)$$