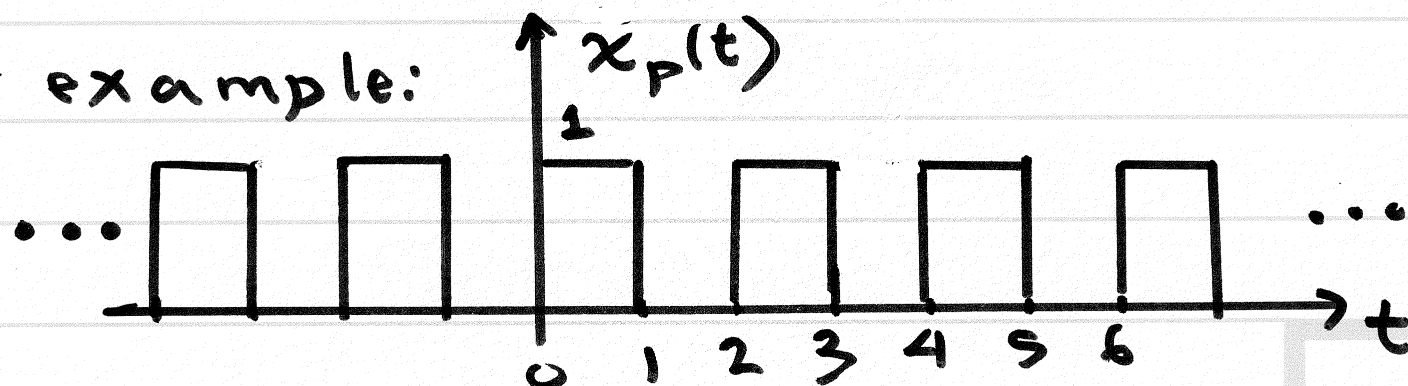
A Solution Approach to Prob. 2.22 (e)

- First, note: any periodic signal with period T can be expressed as

$$x_p(t) = \sum_{k=-\infty}^{\infty} x(t-kT)$$

- For example:



$$x_p(t) = \sum_{k=-\infty}^{\infty} \mathrm{rect}\left(t-\tfrac{1}{2}-k2\right)$$

- where: $\mathrm{rect}\left(t-\frac{1}{2}\right) = u(t)-u(t-1)$

- Consider periodic signal $x_p(t)$ as input to LTI system with impulse response $h(t)$

$$x_p(t) \longrightarrow \boxed{h(t)} \longrightarrow y_p(t)$$

- $y_p(t)$ will also be periodic with period T
- simple to show using superposition/distributive property of convolution and time-invariance

$$y_p(t) = x_p(t) * h(t) = \left\{ \sum_{k=-\infty}^{\infty} x(t-kT) \right\} * h(t)$$

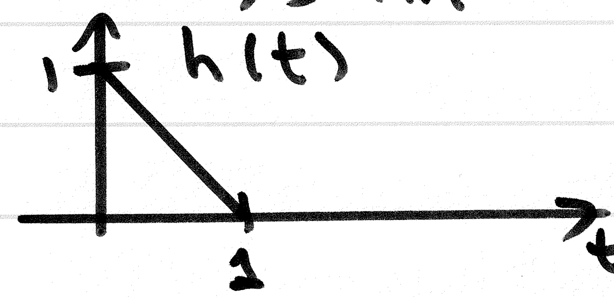
$$= \sum_{k=-\infty}^{\infty} x(t-kT) * h(t)$$

$$= \sum_{k=-\infty}^{\infty} y(t-kT)$$

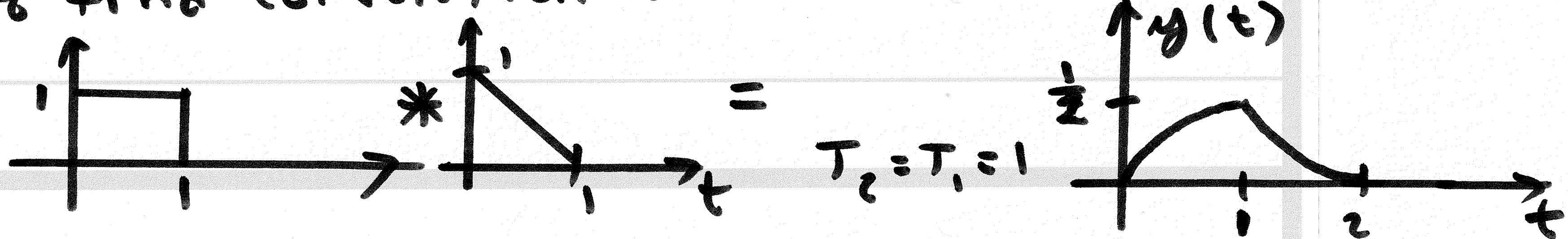
where: $y(t) = x(t) * h(t)$

- So, to find the output $y_p(t)$, you only have to convolve <u>one</u> period of $x_p(t)$ with $h(t)$ to form $y(t) = x(t) * h(t)$ and then repeat $y(t)$ every T secs

- <u>Continuing example:</u> Suppose $x_p(t)$ = periodic train of rectangular pulses was input to LTI system with impulse response



- We can use ramp-down triangle convolution result to find convolution below with no work



$$y(t) = \begin{cases} -\frac{t^2}{2} + t, & 0 < t < 1 \\ +\frac{t^2}{2} - 2t + 2, & 1 < t < 2 \end{cases}$$

concave down

concave up

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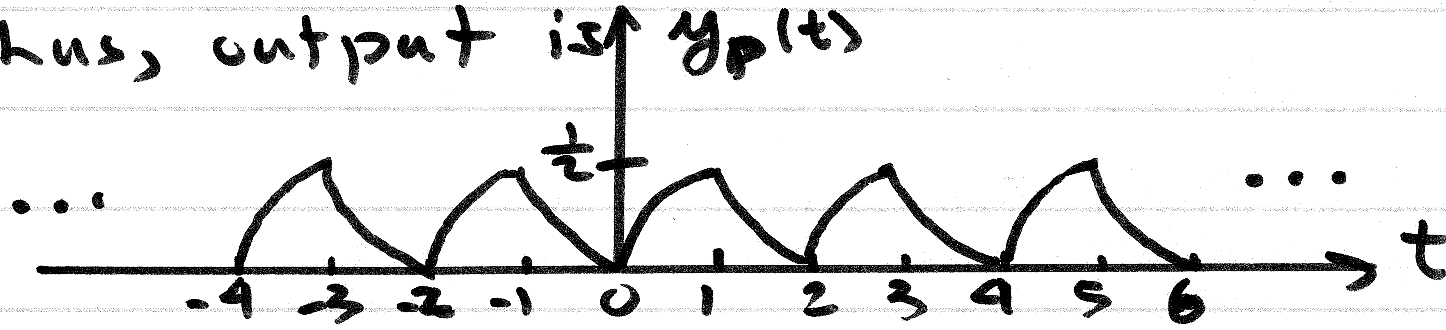
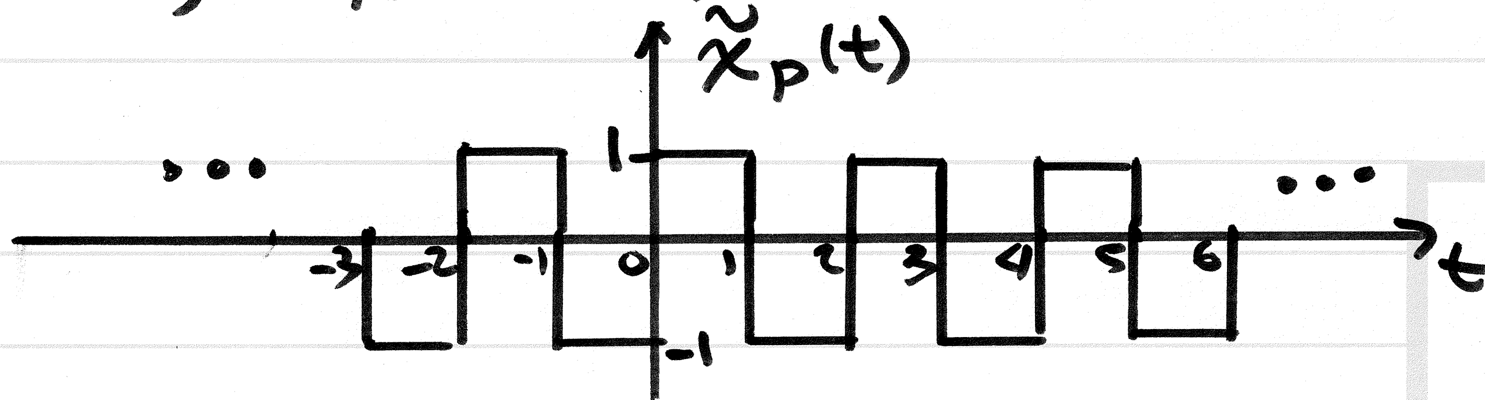
• Thus, output is



• Now, suppose input was instead



• <u>Observe</u>: $\tilde{x}_p(t) = x_p(t) - x_p(t-1)$

where $x_p(t)$ is plotted on pg. 1

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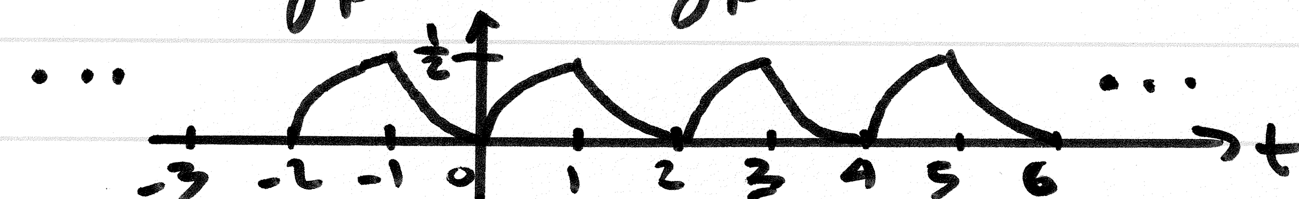
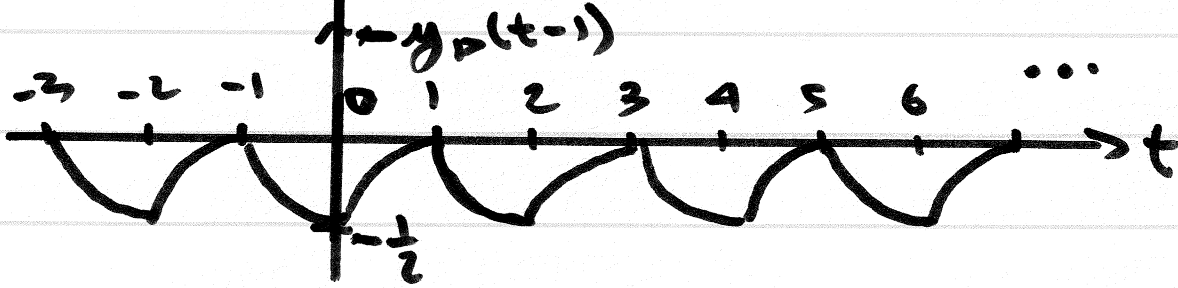
- Invoking LTI: $\tilde{y}_p(t) = \tilde{x}_p(t) * h(t)$

$$\tilde{y}_p(t) = \left(x_p(t) - x_p(t-1)\right) * h(t)$$

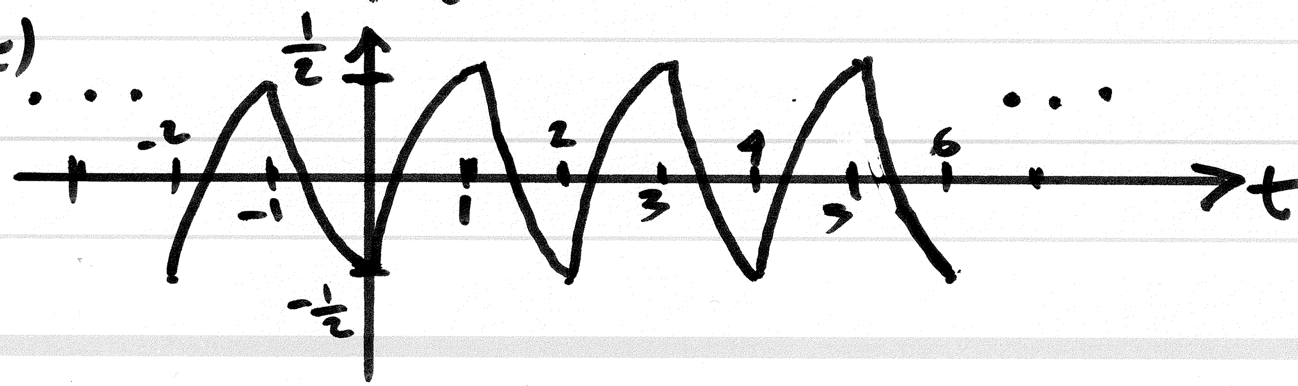
$$= x_p(t) * h(t) - x_p(t-1) * h(t)$$

$$= y_p(t) - y_p(t-1)$$





$\tilde{y}_p(t)$



- The input for Prob. 2.22 (e) was $\tilde{x}_p(t+\frac{1}{2})$

$$z(t) = \tilde{x}_p(t+\tfrac{1}{2}) * h(t)$$ 6

$$= \tilde{y}_p(t+\tfrac{1}{2})$$

- Just take answer $\tilde{y}_p(t)$ at bottom of page 5 and shift to left by 1/2

- This whole problem was solved without having to do a convolution $\Rightarrow$ but rather by using a known convolution result and invoking concepts of linearity and time-invariance