Part (a)

Prob. 2.11

LTI

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$\frac{dx(t)}{dt} \rightarrow \boxed{h(t)} \rightarrow \text{show } z(t) = \frac{dy(t)}{dt}$$

- $x(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \frac{dx(t)}{dt}$ is an LTI system

- Thus: $x(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \boxed{h(t)} \rightarrow z(t)$

Part (b)

- Invoking associativity & commutativity:

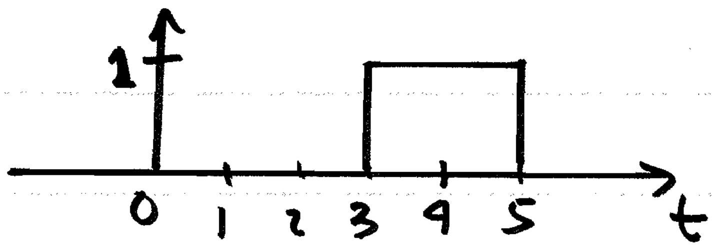
$$x(t) \rightarrow \boxed{h(t)} \xrightarrow{y(t)} \boxed{\frac{d}{dt}} \rightarrow z(t)$$

Part (c)

Thus: $z(t) = \frac{dy(t)}{dt}$ where: $y(t) = x(t) * h(t)$

Point of problem is to show answer to (c) is identical to answer to (b)
AS the order of two LTI Systems in series does not matter (commutativity)
and d/dt is an LTI System

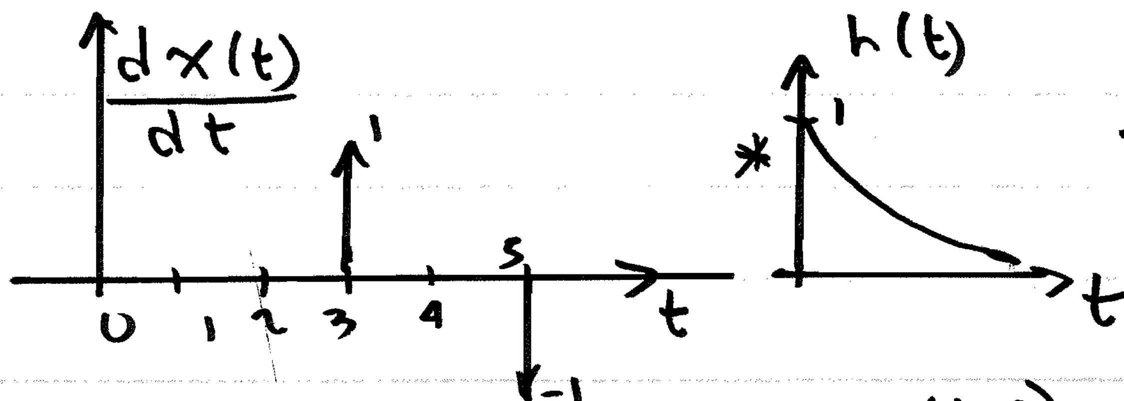
$$x(t) = u(t-3) - u(t-5) = \text{rect}\left(\frac{t-4}{2}\right) \qquad h(t) = e^{-3t}u(t)$$



• Suppose you were interested in $z(t) = \dfrac{d\,y(t)}{dt}$

where $y(t) = x(t) * h(t)$

• It's certainly easier to first differentiate $x(t)$ and then convolve with $h(t)$:



$= z(t)$

Part (b) answer below

$$z(t) = h(t-3) - h(t-5) = e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$$

$$= \frac{d}{dt}y(t) \qquad \text{where: } y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

Previous page was solution to part (b)

Now, let's do part (a)

$y(t) = x(t) * h(t)$ $\qquad$ $x(t) = u(t-3) - u(t-5)$

$h(t) = e^{-3t}u(t)$

First:

$$e^{-3t}u(t) * \{u(t) - u(t-2)\}$$

$$= e^{-3t}u(t) * u(t) - e^{-3t}u(t) * u(t-2)$$

$$= \left(\frac{1}{-3-0}e^{-3t} + \frac{1}{0-(-3)}e^{0t}\right)u(t) -$$

$$= \frac{1}{3}\left(1 - e^{-3t}\right)u(t) - \frac{1}{3}\left(1 - e^{-3(t-2)}\right)u(t-2)$$

Then: ans to part (a) $\qquad$ Answer to (a)

$$y(t) = \frac{1}{3}\left(1 - e^{-3(t-3)}\right)u(t-3) - \frac{1}{3}\left(1 - e^{-3(t-5)}\right)u(t-5)$$

(c) derivative of this ans for (a) should be ans to (b)

· have to use product rule:

$$\frac{d}{dt}y(t) = \underbrace{\frac{1}{3}\left(1-e^{-3(t-3)}\right)\delta(t-3)}_{=0}$$

Sifting Prop. of Delta Fn.

$$- (-3)\frac{1}{3}e^{-3(t-3)}\,u(t-3)$$

$$\underbrace{-\frac{1}{3}\left(1-e^{-3(t-5)}\right)\delta(t-5)}_{=0}$$

$$+ (-3)\frac{1}{3}e^{-3(t-5)}\,u(t-5)$$

$$= e^{-3(t-3)}u(t-3) - e^{-3(t-5)}u(t-5)$$