Prob. 1.20 Given system is Linear and two Input/Output pairs:

$$x_1(t) = e^{j2t} \longrightarrow \boxed{S} \longrightarrow y_1(t) = e^{j3t}$$

$$x_2(t) = e^{-j2t} \longrightarrow \boxed{S} \longrightarrow y_2(t) = e^{-j3t}$$

Since: $x(t) = \cos(2t) = \frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t}$

$$= \frac{1}{2} x_1(t) + \frac{1}{2} x_2(t)$$

and system is linear, it follows that

$$y(t) = \frac{1}{2} y_1(t) + \frac{1}{2} y_2(t) = \frac{1}{2} e^{j3t} + \frac{1}{2} e^{-j3t}$$

$$= \cos(3t)$$

- Next, consider time-shifting $x(t)$ as input:

$$w(t) = \cos\left(2\left(t - \tfrac{1}{2}\right)\right) \longrightarrow \boxed{S} \longrightarrow z(t) = ?$$

$$= x(t - \tau)$$

$$\tau = 1/2$$

- <u>Not</u> told $S$ was TI.

In fact, since we input sinewave with frequency of 2 rads/sec and got an output sinewave with frequency of 3 rads/sec, you can immediately surmise the system is not both Linear <u>and</u> Time-Invariant (not LTI)

- If a system is LTI, it's easy to prove:

$$x(t)=A_i e^{j\theta_i} e^{j\omega_0 t} \longrightarrow \boxed{\begin{array}{c}\text{LTI}\\ \text{System}\end{array}} \longrightarrow y(t)=A_o e^{j\theta_o} e^{j\omega_0 t}$$

- you get a sineware at the output with same frequency as input sinewave, only with ~~possibly~~ generally a new amplitude and phase

- Since system is linear, but gave us $y(t)=e^{j3t}$ for the input $x(t)=e^{j2t}$

we can immediately surmise that the system is <u>not</u> TI (not Time Invariant)

- Thus:

$$w(t) = \cos\left(2\left(t - \tfrac{1}{2}\right)\right) = \cos(2t - 1)$$

$$= \frac{1}{2} e^{j2t} e^{-j} + \frac{1}{2} e^{-j2t} e^{j}$$

$$= \left(\frac{1}{2} e^{-j}\right) e^{j2t} + \left(\frac{1}{2} e^{j}\right) e^{-j2t}$$

Thus:

$$z(t) = \frac{1}{2} e^{-j} e^{j3t} + \frac{1}{2} e^{j} e^{-j3t}$$

$$= \frac{1}{2} e^{j\,3\left(t - \frac{1}{3}\right)} + \frac{1}{2} e^{-j\,3\left(t - \frac{1}{3}\right)}$$

$$= \cos\left(3\left(t - \tfrac{1}{3}\right)\right)$$

- If the system had been LTI, the output would have been $\cos\left(3\left(t - \frac{1}{2}\right)\right)$