

Prob. 1.20 Given system is Linear and two Input/Output pairs:

$$x_1(t) = e^{j2t} \rightarrow \boxed{\mathcal{S}} \rightarrow y_1(t) = e^{j3t}$$

$$x_2(t) = e^{-j2t} \rightarrow \boxed{\mathcal{S}} \rightarrow y_2(t) = e^{-j3t}$$

Since: $x(t) = \cos(2t) = \frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t}$
 $= \frac{1}{2} x_1(t) + \frac{1}{2} x_2(t)$

and system is linear, it follows that

$$y(t) = \frac{1}{2} y_1(t) + \frac{1}{2} y_2(t) = \frac{1}{2} e^{j3t} + \frac{1}{2} e^{-j3t} = \cos(3t)$$

- Next, consider time-shifting $x(t)$ as input: ②

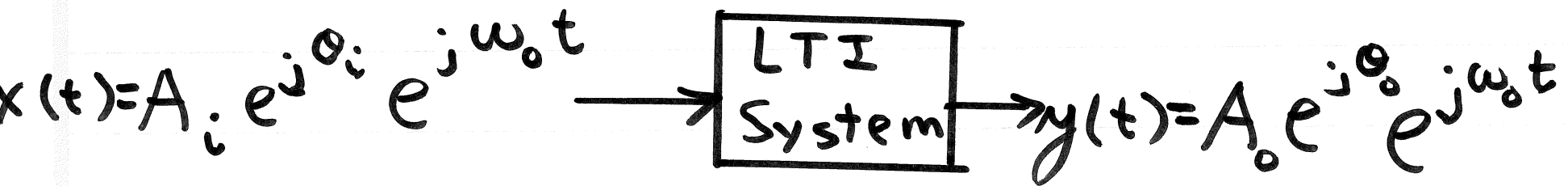
$$w(t) = \cos\left(2\left(t - \frac{1}{2}\right)\right) \rightarrow \boxed{S'} \rightarrow z(t) = ?$$
$$= x(t - \tau)$$
$$\tau = 1/2$$

- Not told S' was TI.

In fact, since we input sine wave with frequency of 2 rads/sec and got an output sine wave, frequency of 3 rads/sec, you can immediately surmise the system is not both Linear and Time-Invariant (not LTI)

• If a system is LTI, it's easy to prove:

③



• You get a sine wave at the output with same frequency as input sine wave, only with ~~possibly~~ generally a new amplitude and phase

• Since system is linear, but gave us $y(t) = e^{j3t}$ for the input $x(t) = e^{j2t}$

we can immediately surmise that the system is not TI (not Time Invariant)

• Thus:

④

$$\begin{aligned}w(t) &= \cos\left(2\left(t - \frac{1}{2}\right)\right) = \cos(2t - 1) \\ &= \frac{1}{2} e^{j2t} e^{-j} + \frac{1}{2} e^{-j2t} e^j \\ &= \left(\frac{1}{2} e^{-j}\right) e^{j2t} + \left(\frac{1}{2} e^j\right) e^{-j2t}\end{aligned}$$

Thus:

$$\begin{aligned}z(t) &= \frac{1}{2} e^j e^{j3t} + \frac{1}{2} e^{-j} e^{-j3t} \\ &= \frac{1}{2} e^{j3\left(t - \frac{1}{3}\right)} + \frac{1}{2} e^{-j3\left(t - \frac{1}{3}\right)} \\ &= \cos\left(3\left(t - \frac{1}{3}\right)\right)\end{aligned}$$

- If the system had been LTI, the output would have been $\cos\left(3\left(t - \frac{1}{2}\right)\right)$