Practice Problems for Exam 2

1

$$x(t) = \frac{-1}{2}\left\{\frac{\sin\left(10\left(t-\frac{\pi}{10}\right)\right)}{\pi(t-10)} - \frac{\sin\left(10\left(t+\frac{\pi}{10}\right)\right)}{\pi(t+10)}\right\}$$

$$X(\omega) = -\operatorname{rect}\left(\frac{\omega}{20}\right)\left\{\frac{1}{2}e^{-j\omega\frac{\pi}{10}} - \frac{1}{2}e^{j\omega\frac{\pi}{10}}\right\}$$

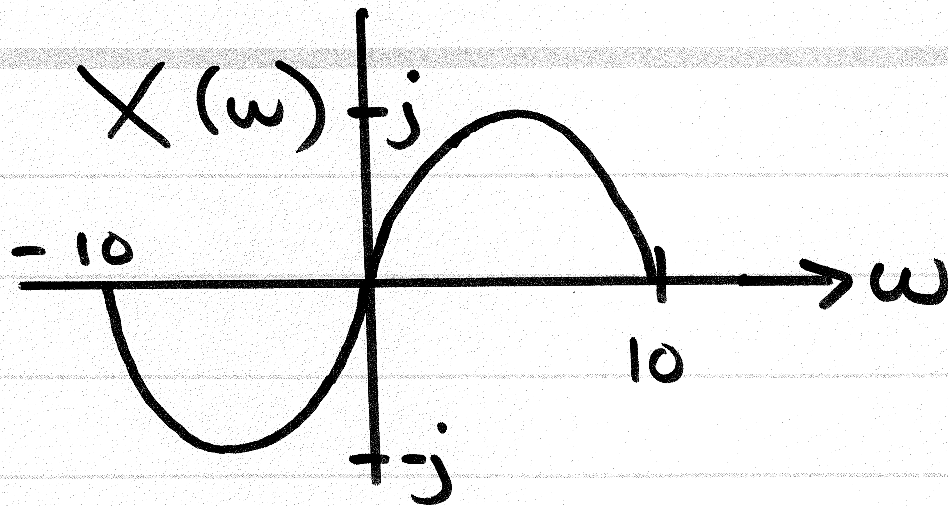
· Using: $\dfrac{\sin(Wt)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} \operatorname{rect}\left(\dfrac{\omega}{2W}\right)$

and $x(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_0}X(\omega)$

Thus: $X(\omega) = j\sin\left(\pi\frac{\omega}{10}\right)\operatorname{rect}\left(\frac{\omega}{20}\right)$

purely imaginary since $x(-t) = -x(t)$

odd function



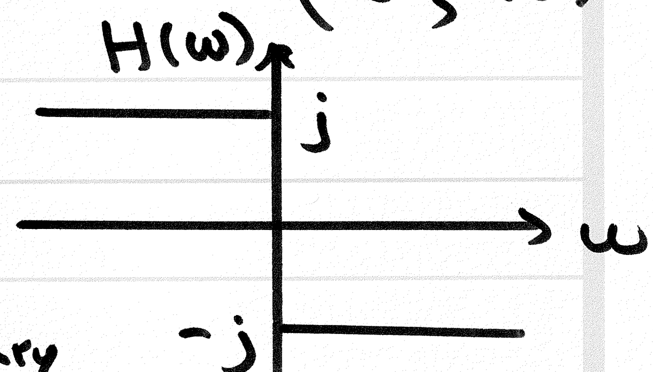
• Next, create $\tilde{x}(t) = x(t) + j\hat{x}(t)$

where: $\hat{x}(t) = x(t) * h(t) \xleftrightarrow{\mathcal{F}} \hat{X}(\omega) = X(\omega)H(\omega)$

with:

$$h(t) = \frac{1}{\pi t} \xleftrightarrow{\mathcal{F}} H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$

odd function in time $\Rightarrow$ purely imaginary in frequency

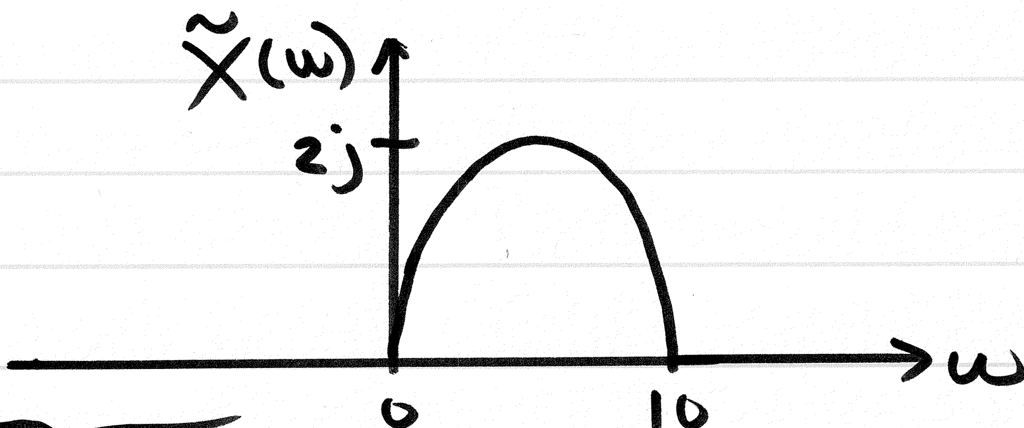


Take FT of $\tilde{x}(t) = x(t) + j\,\hat{x}(t)$

$$\tilde{X}(\omega) = X(\omega) + j\,\hat{X}(\omega)$$

$$= \begin{cases} X(\omega) + j\,(j X(\omega)) & \omega < 0 \\ X(\omega) + j\,(-j X(\omega)) & \omega > 0 \end{cases}$$

$$= \begin{cases} 0, & \omega < 0 \\ 2X(\omega), & \omega > 0 \end{cases}$$



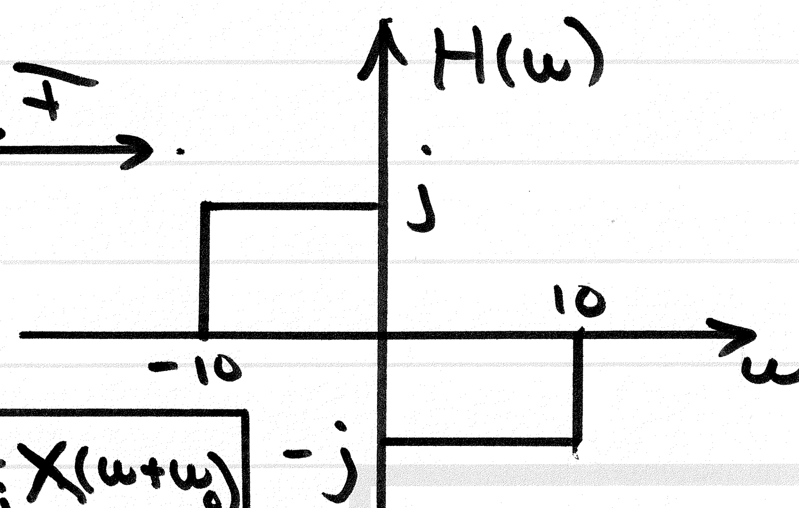
no negative frequency content

• Consider: $\tilde{x}(t) = x(t) + j\,\hat{x}(t)$ 4

where: $\hat{x}(t) = x(t) * h(t)$

where:

$h(t) = \dfrac{2\sin(5t)}{\pi t}$ $\quad \sin(5t) \xleftrightarrow{\mathcal{F}}$.



recall:

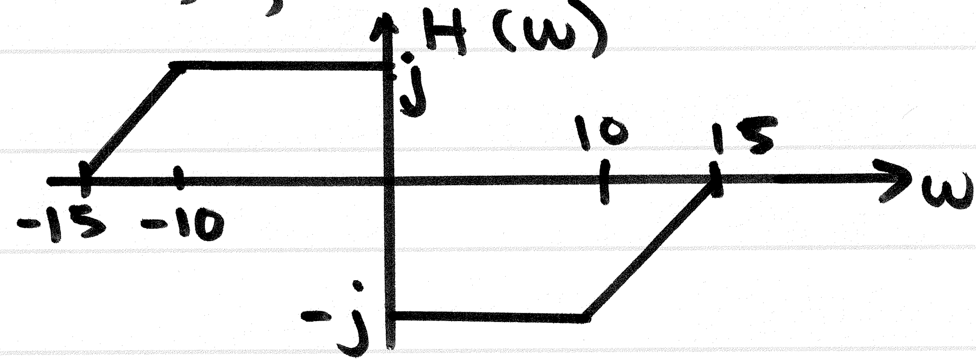
$$\boxed{x(t)\sin(\omega_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0)}$$

Would $\tilde{X}(\omega)$ be any different from before for the original signal $x(t)$ on pg 1 ??

No. They would be the same.

• How about if we created $\hat{x}(t)$ as (5)

$\hat{x}(t) = x(t) * h(t)$, where:

$h(t) \overset{\mathcal{F}}{\longleftrightarrow}$ 

$\tilde{x}(t) = x(t) + j\hat{x}(t)$ would still be the same with no negative frequency content

Next, plot the Fourier Transform of

$y(t) = x(t)\cos(25t) - \hat{x}(t)\sin(25t)$

Where did this come from?

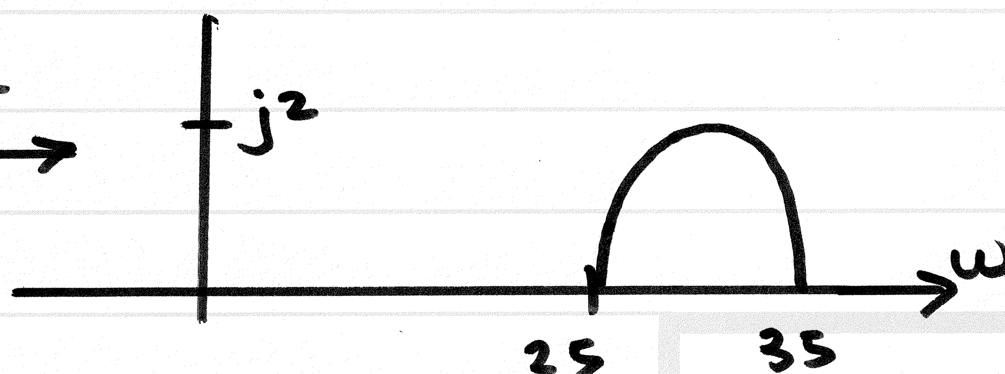
$$y(t) = \mathrm{Re}\left\{ \tilde{x}(t)\, e^{j25t} \right\}$$

$$= \mathrm{Re}\left\{ \left(x(t) + j\hat{x}(t)\right)\left(\cos(25t) + j\sin(25t)\right) \right\}$$

- Now, recall: $e^{j\omega_0 t}\,\tilde{x}(t) \xleftrightarrow{\mathcal{F}} \tilde{X}(\omega - \omega_0)$

- Thus:

$$\tilde{x}(t)\, e^{j25t} \xleftrightarrow{\mathcal{F}}$$



- What does taking the real part of a signal in time do in the frequency domain?
  - since $x^*(t) \xleftrightarrow{\mathcal{F}} X^*(-\omega)$
- then: $\mathrm{Re}\{x(t)\} = \frac{1}{2}x(t) + \frac{1}{2}x^*(t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}X(\omega) + \frac{1}{2}X^*(-\omega)$

Thus, the Fourier Transform of

$$y(t) = x(t)\cos(25t) - \hat{x}(t)\sin(25t)$$

is: