LTI System Properties Determined from Impulse Response

$$x(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI}\\ \text{System}\end{array}} \longrightarrow y(t)$$

$$\delta(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI}\\ \text{System}\end{array}} \longrightarrow h(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

System is causal if $h(t) = 0$ for $t < 0$

stable iff $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

memoryless iff $h(t) = K\,\delta(t)$

invertible if there exists an LTI (inverse) system with impulse response $h_I(t)$ that satisfies

$$h(t) * h_I(t) = \delta(t)$$

Discrete-Time LTI Systems

$$x[n] \longrightarrow \boxed{\text{LTI System}} \longrightarrow y[n]$$

$$\delta[n] \longrightarrow \boxed{\text{LTI System}} \longrightarrow h[n]$$

Kronecker Delta

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

System is causal if $h[n] = 0$ for $n < 0$

System is stable iff $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

memoryless iff $h[n] = K\,\delta[n]$

Invertible if there exists an LTI inverse system with impulse response $h_I[n]$ satisfying

$$h[n] * h_I[n] = \delta[n]$$