Interpretation of the Fourier Transform

- Recall for $x(t)$ periodic with period T:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

$\Delta\omega = 2\pi/T$

$=$ sum of sinewaves equi-spaced in frequency

$\Rightarrow$ spacing between frequencies $= \frac{2\pi}{T} = \Delta\omega$

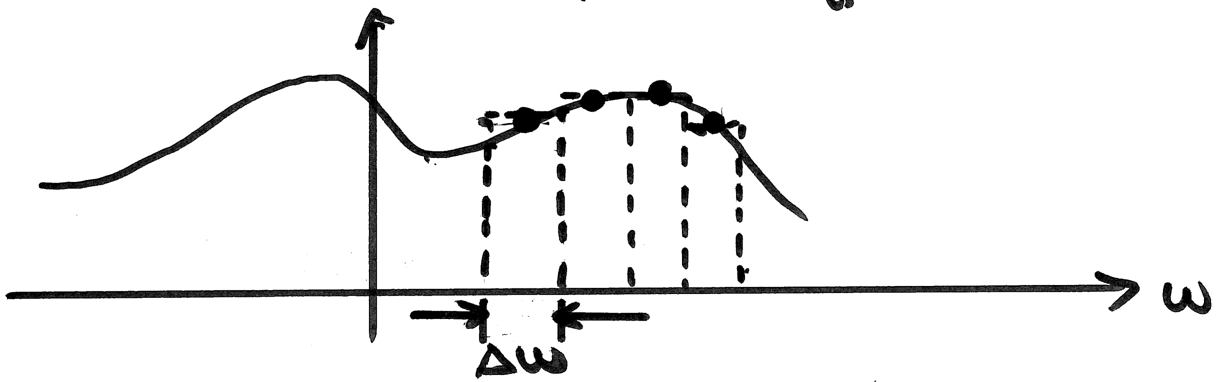
$\Rightarrow$ Fourier Series coefficients are the complex amplitudes a_k

- For aperiodic signal, Inverse Fourier Transform:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

- Approximate "area under the curve" as area under a bunch of rectangles of width $\Delta\omega$

$$x(t) \approx \sum_{k=-\infty}^{\infty} \underbrace{\tfrac{1}{2\pi} X(k\Delta\omega) e^{jk\Delta\omega t}}_{\text{height of } k\text{-th rectangle}} \; \underbrace{\Delta\omega}_{\text{width}}$$



- Rearranging:

$$x(t) \approx \sum_{k=-\infty}^{\infty} \underbrace{\frac{\Delta\omega}{2\pi} X(k\Delta\omega)}_{a_k} \; \underbrace{e^{jk\Delta\omega t}}_{\text{sum of sinewaves equi-spaced by } \Delta\omega \text{ in frequency}}$$

$$x(t) \approx \sum_{k=-\infty}^{\infty} \underbrace{\frac{\Delta\omega}{2\pi} X(k\Delta\omega)}_{a_k} \underbrace{e^{jk\Delta\omega t}}_{\substack{\text{sum of sinewaves} \\ \text{equi-spaced by} \\ \Delta\omega \text{ in frequency}}}$$

- as $\Delta\omega \to d\omega$, get back to Inverse FT:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

- so, Inverse FT can be interpreted as expressing an aperiodic (not periodic) signal as a sum of an infinite number of sinewaves that are equi-spaced <u>infinitesimally</u> close in frequency
- and the Fourier Transform is the distribution of the complex amplitudes of all those sinewaves

Calrulation of Fourier Transform:

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt = \sum_{n=-\infty}^{\infty} x(n\Delta t)\, e^{-jw n\Delta t}\, \Delta t$$

- Evaluate at equi-spaced frequencies:

$$X(k\Delta w) = \sum_{n=-N}^{N} x(n\Delta t)\, e^{-jk\Delta w\, n\Delta t}\, \Delta t$$