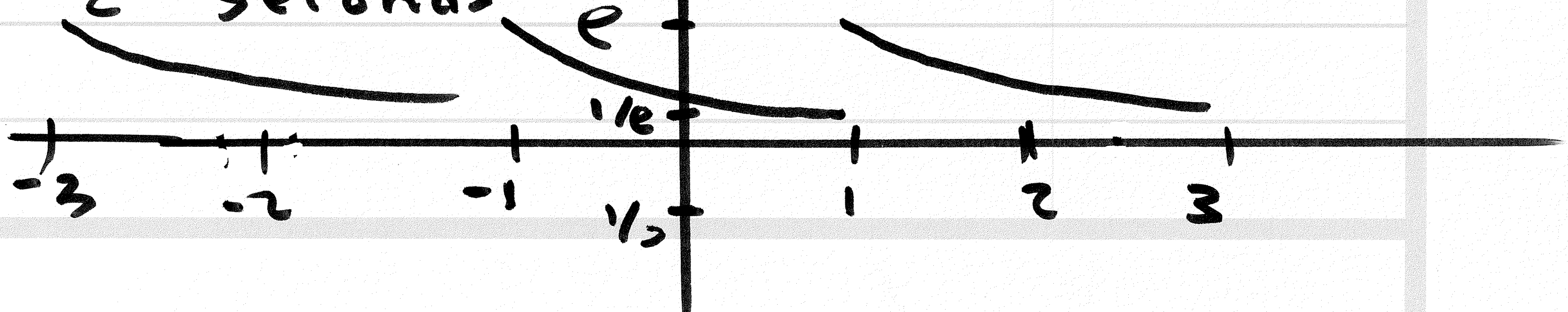
Help with Hmwk #5 ⇒ Chap. 3 Problems on Fourier Series for Periodic Signals

- For almost all problems, use basic Fourier Series for periodic train of rectangular pulses or Delta Functions in conjunction with properties listed in Table 3.1 on page 206.

- Prob. 3.22 (b) is only problem you should do the integral to find the F.S. coefficients

$x(t) = e^{-t}$ for $-1 < t < 1$, repeats every 2 seconds.



$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$ ②

$$= \frac{1}{2}\int_{-1}^{1} e^{-t}\, e^{-jk\frac{2\pi}{2}t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\pi)t}\, dt$$

$$= \frac{1}{2}\frac{1}{-(1+jk\pi)}\, e^{-(1+jk\pi)t}\Big]_{-1}^{1}$$

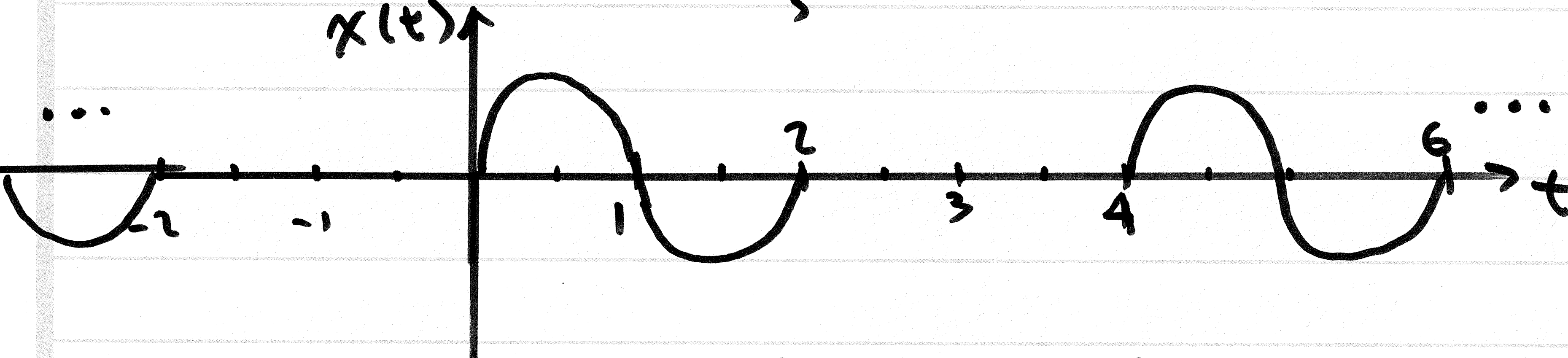
$$= \frac{-1}{2(1+jk\pi)}\left\{e^{-(1+jk\pi)} - e^{+(1+jk\pi)}\right\}$$

$$= \frac{-1}{2(1+jk\pi)}\left\{e^{-1}(-1)^k - e\,(-1)^k\right\}$$

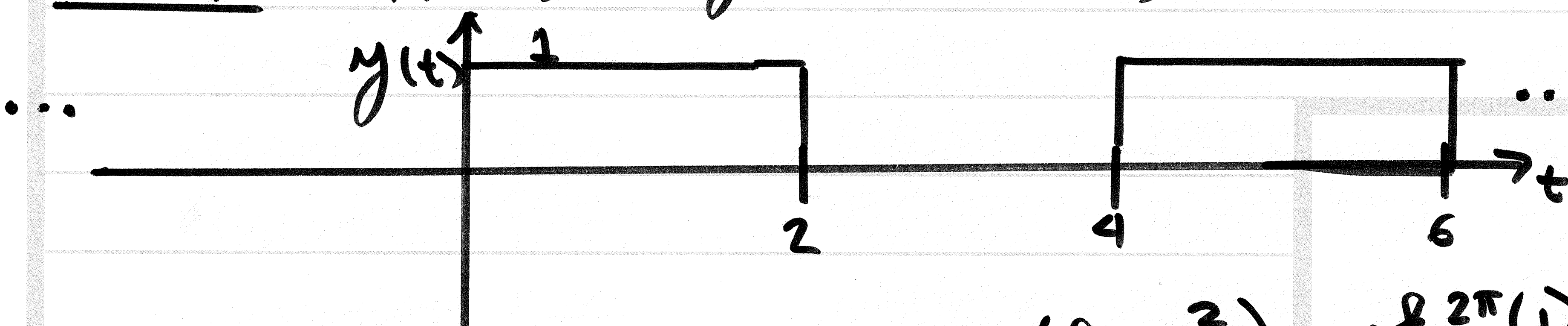
$$= \frac{1}{2(1+jk\pi)}(-1)^k\left(e - \frac{1}{e}\right)$$

Prob. 3.22 (c)

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 \le t \le 4 \end{cases} \qquad \text{period} = T = 4$$



<u>TRICK</u> : $x(t) = y(t)\sin(\pi t)$ = where:



F.S. for y(t) above: $b_k = \dfrac{\sin\left(k\pi \frac{2}{4}\right)}{k\pi} e^{-jk\frac{2\pi}{4}(1)}$

$$b_k = \frac{\sin(k\frac{\pi}{2})}{k\pi}(-j)^k \qquad \boxed{j = e^{j\frac{\pi}{2}}}$$

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Note: $\sin(\pi t) = \sin\left(2\pi \frac{2}{4} t\right)$

$$= \frac{1}{2j} e^{j 2\pi \frac{(2)}{4} t} - \frac{1}{2j} e^{+j 2\pi \frac{(-2)}{4} t}$$

Also: Frequency Shift Prop. from Table 3.1

$x(t) \rightarrow$ F.S. a_k

then $x(t) e^{j 2\pi \frac{m}{T} t} \longrightarrow$ F.S. a_{k-m}

Proof: $\sum_{k=-\infty}^{\infty} a_k e^{j 2\pi \frac{k}{T} t} \; e^{j 2\pi \frac{m}{T} t} = \sum_{k=-\infty}^{\infty} a_k e^{j 2\pi \frac{(k+m)}{T} t}$

change of variables $k' = k + m$

$\Rightarrow k = k' - m$

$$= \sum_{k'=-\infty}^{\infty} a_{k'-m} e^{j 2\pi \frac{k'}{T} t}$$

• Thus: $x(t) = y(t)\left\{ \frac{1}{2j} e^{j\frac{2\pi(2)}{4}t} - \frac{1}{2j} e^{j\frac{2\pi(-2)}{4}t} \right\}$ $\quad m=2 \quad m=-2$

and

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$$a_k = \frac{1}{2j} b_{k-2} - \frac{1}{2j} b_{k+2}$$

$$= \frac{1}{2j} \frac{\sin\left[(k-2)\frac{\pi}{2}\right]}{(k-2)\pi} (-j)^{k-2} - \frac{1}{2j} \frac{\sin\left[(k+2)\frac{\pi}{2}\right]}{(k+2)\pi} (-j)^{k+2}$$

= -2 j (-j)^k sin(k π /2) / π (k^2 -4)

Prob. 3.40 $\quad x(t) \longrightarrow$ F.S. a_k

(a) $y(t) = x(t-t_0) + x(t+t_0) \qquad b_k = ?$

$$b_k = a_k e^{-j 2\pi \frac{k}{T} t_0} + a_k e^{-j 2\pi \frac{k}{T}(-t_0)}$$

$$= 2 a_k \cos\left(2\pi \frac{k}{T} t_0\right)$$

(b) $y(t) = \mathcal{E}v\{x(t)\} = \frac{1}{2} x(t) + \frac{1}{2} \underbrace{x(-t)}$

see Table 3.1 time-reverse property

$$b_k = \frac{1}{2} a_k + \frac{1}{2} a_{-k}$$

(c) $y(t) = Re\{x(t)\} = \frac{1}{2} x(t) + \frac{1}{2} \underbrace{x^*(t)}$

see Table 3.1 conjugation property

$$b_k = \frac{1}{2} a_k + \frac{1}{2} a^*_{-k}$$

(d) $y(t) = \frac{d^2}{dt^2} x(t)$ 7

$$b_k = \left(jk\frac{2\pi}{T}\right)^2 a_k = -\left(k\frac{2\pi}{T}\right)^2 a_k$$

(e) $y(t) = x(3t-1) = x\left(3\left(t-\frac{1}{3}\right)\right)$

First: $z(t) = x(3t)$

Time-Scaling does not change F.S. coeffts. just the period

Here, new period for $z(t)$ is $\frac{T}{3}$

Then: $y(t) = z\left(t-\frac{1}{3}\right)$

Thus: $b_k = a_k e^{-j\frac{2\pi k}{T/3}\left(\frac{1}{3}\right)} = a_k e^{-j2\pi\frac{k}{T}}$