Example. Prob. 2.26 $y[n] = x_1[n] * x_2[n] * x_3[n]$

$x_1[n] = \left(\frac{1}{2}\right)^n u[n]$ $x_2[n] = u[n+3]$ $x_3[n] = \delta[n] - \delta[n-1]$

(a) $x_1[n] * x_2[n] = ? = z[n]$

Use standard convolution result for α^n u[n] * β^n u[n] with β=1
then use time-invariance and replace n by n+3 everywhere

$\alpha = \frac{1}{2}$ $\beta = 1$

$c_1 = \frac{\beta}{\beta - \alpha} = \frac{1}{1-\frac{1}{2}} = 2$ $c_2 = \frac{\alpha}{\alpha - \beta} = \frac{1/2}{1/2 - 1} = -1$

$\underbrace{x_1[n] * x_2[n]}_{} = 2\, u[n+3] - \left(\frac{1}{2}\right)^{n+3} u[n+3]$

(b) $z[n] * x_3[n]$ ⇒ easiest way

$z[n] * x_3[n] = z[n] * \{\delta[n] - \delta[n-1]\}$

$= z[n] - z[n-1]$

$= 2\, u[n+3] - \left(\frac{1}{2}\right)^{n+3} u[n+3] - 2\, u[n+2] + \left(\frac{1}{2}\right)^{n+2} u[n+2]$

$= \delta[n+3] + \left(-\frac{1}{8} + \frac{1}{4}\right)\left(\frac{1}{2}\right)^n u[n+2]$

$= \delta[n+3] + {}^{1}/8 \left(\frac{1}{2}\right)^n u[n+2]$

You do parts (c) and (d)

Prob. 2.26 (cont.)

Ans to (b) further simplifies as

$$z[n] = \frac{1}{8}\left(\frac{1}{2}\right)^n u[n+3] = \left(\frac{1}{2}\right)^{n+3} u[n+3]$$

Now (c) and (d):

First convolve $x_2[n]$ and $x_3[n]$

$$u[n+3] * (\delta[n] - \delta[n-1])$$

$$= u[n+3] * \delta[n] - u[n+3] * \delta[n-1]$$

$$= u[n+3] - u[n+2] = \delta[n+3]$$

Next, convolve this with $x_1[n]$

$$\left(\frac{1}{2}\right)^n u[n] * \delta[n+3] = \left(\frac{1}{2}\right)^{n+3} u[n+3]$$

- Demonstrates associativity of DT convolution:

$$(x_1[n] * x_2[n]) * x_3[n] = x_1[n] * (x_2[n] * x_3[n])$$

Example. Prob. 2.5 $\quad y[n] = x[n] * h[n]$

$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases} = u[n] - u[n-10]$$

$$h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{otherwise} \end{cases} = u[n] - u[n-(N+1)]$$

Find N given $y[4] = 5 \quad y[14] = 0$

From convolution of two DT rectangles result on bottom of pg. 2:

length of $y[n] = 10 + (N+1) - 1 = N + 10$

is nonzero from $n=0$ to $n = N+10-1 = N+9$

So: $N + 9 < 14$ so $N < 5$

So $N_1 = N$ and $N_2 = 9$ at bottom of pg. 2

Check $N = 4 = N_1$, $\Rightarrow y[N_1] = N_1 + 1 \Rightarrow y[4] = 5$ ✓ checks

Answer: $N = 4$

Prob. 2.21 part (d) $\quad y[n] = x[n] * h[n]$

$$x[n] = u[n] - u[n-5] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Define:

$$\tilde{h}[n] = u[n] - u[n-6] = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$$

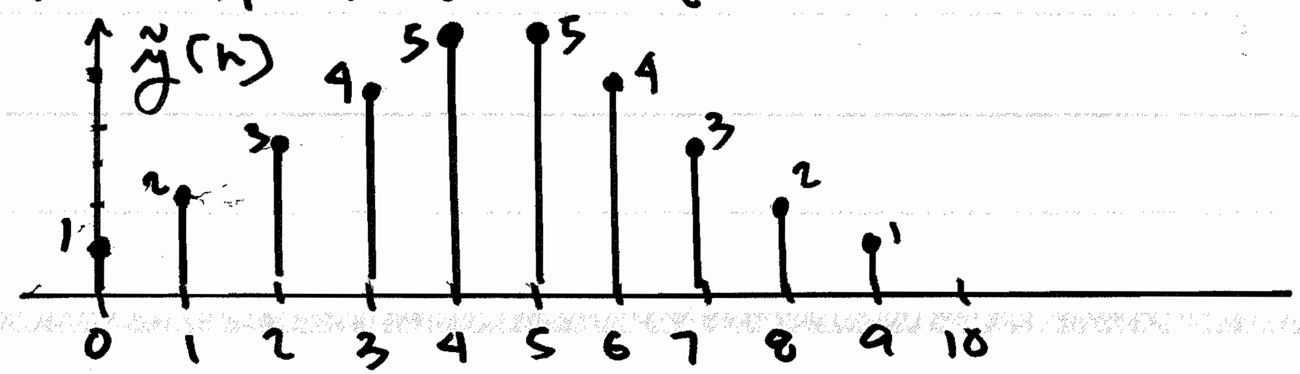
Then: $h[n] = \tilde{h}[n-2] + \tilde{h}[n-11]$

It follows: define $\tilde{y}[n] = x[n] * \tilde{h}[n]$

Then:

$$y[n] = \tilde{y}[n-2] + \tilde{y}[n-11]$$

where: $\tilde{y}[n]$ determined from bottom of pg. 2 with $N_1 = 4$ and $N_2 = 5$



$\tilde{y}[n] = \{1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

$\uparrow$
$n=0$

$y[n] = \tilde{y}(n-2) + \tilde{y}[n-11]$

n: 0 1 2 3 4 5 6 7 8 9 10 11

$= \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

$+ \{$ $1, 2, 3, 4, 5, 5, 4, 3, 2, 1$

Final answer:

$y[n] = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 2, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

$\uparrow$
$n=0$

single point where the two terms overlap at $n=11$