

## Example

Hmwk. Prob. 1.18

(6)

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

• This is a difference eqn. It's just non-recursive ( $a_k=0$ ) and non-causal } see pg (7A)

• So it's LTI. But we'll prove it anyhow

(a) Linear? Inputting  $x_1[n]$  and  $x_2[n]$  individually we have:

$$\textcircled{1} y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$$

$$\textcircled{2} y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$\alpha_1 \textcircled{1} + \alpha_2 \textcircled{2} \Rightarrow$$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \alpha_1 \sum_{k=n-n_0}^{n+n_0} x_1[k] + \alpha_2 \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Rearranging:

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{k=n-n_0}^{n+n_0} \left\{ \alpha_1 x_1[k] + \alpha_2 x_2[k] \right\}$$

If the system is linear, that's exactly the equation we would obtain if we input

$x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$  into the system

i.e., if  $S$  is linear:

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] \rightarrow \boxed{S} \rightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

• which, in turn, dictates:

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{k=n-n_0}^{n+n_0} \left( \alpha_1 x_1[k] + \alpha_2 x_2[k] \right)$$

and we already proved that this eqn. holds

$\Rightarrow$  system is linear

Side-note:  $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$

7a

• change of variables:  $k' = -n + k$   
 $k' = k - n$

new limits on sum:

$$k \Big]_{n-n_0}^{n+n_0} \Rightarrow k' \Big]_{n-n_0-n=-n_0}^{n+n_0-n=n_0}$$

• Substitute:  $k = k' + n$

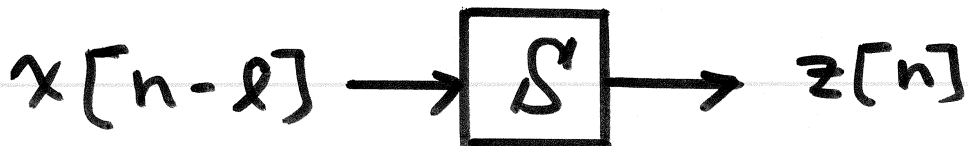
$$y[n] = \sum_{k'=-n_0}^{n_0} x[n+k']$$

This is a non-causal difference eqn.

You could do another change of variables  $k = -k'$

to obtain  $y[n] = \sum_{k=-n_0}^{n_0} x[n-k]$

• Is System TI? First, note: since  $n_0$  is used in system equation, use  $l$  for time-shift in input signal:



Is  $z[n] = y[n-l]$ ?

$$z[n] = \sum_{k=n-n_0}^{n+n_0} x[k-l]$$

change of variables:  $k' = k - l$

new limits:

$$\left. \begin{array}{l} k \\ \hline \end{array} \right]_{n-n_0}^{n+n_0} \Rightarrow \left. \begin{array}{l} k' \\ \hline \end{array} \right]_{n-n_0-l}^{n+n_0-l}$$

substitute:  $\boxed{k = k' + l}$

$$z[n] = \sum_{k=n-l-n_0}^{n-l+n_0} x[k']$$

Recall:

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

Thus:

$$y[n-l] = \sum_{k=n-l-n_0}^{n-l+n_0} x[k]$$

⑨

Thus:  $z[n] = y[n-l] \Rightarrow$  system is TI

Part (c): Is system stable? Yes

• as long as  $x[n] < B$  for all  $n$ , where  $B < \infty$

• Then the "worst case" or largest value  $y[n]$  can be if  $x[k] = B$  within the window  $n-n_0 \leq k \leq n+n_0$

• Thus,  $y[n] < (2n_0+1)B$

$\Rightarrow$  system is stable