Example

Hmwk. Prob. 1.18

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$$

- This is a difference eqn. It's just non-recursive ($a_k = 0$) and non-causal } see pg. 7a

- So it's LTI. But we'll prove it anyhow

(a) Linear? Inputting $x_1[n]$ and $x_2[n]$ individually we have:

1 $y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$ 2 $y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

α_1 1 $+ \alpha_2$ 2 $\Rightarrow$

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \alpha_1 \sum_{k=n-n_0}^{n+n_0} x_1[k] + \alpha_2 \sum_{k=n-n_0}^{n+n_0} x_2[k]$$

Rearranging:

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{k=n-n_0}^{n+n_0} \left\{ \alpha_1 x_1[k] + \alpha_2 x_2[k] \right\}$$

If the system is linear, that's exactly the equation we would obtain if we input $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$ into the system

i.e., if S is linear:

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \boxed{S} \longrightarrow \alpha_1 y_1[n] + \alpha_2 y_2[n]$$

• which, in turn, dictates:

$$\alpha_1 y_1[n] + \alpha_2 y_2[n] = \sum_{k=n-n_0}^{n+n_0} \left(\alpha_1 x_1[k] + \alpha_2 x_2[k] \right)$$

and we already proved that this eqn. holds

$\Rightarrow$ system is linear

Side-note: $$y[n]=\sum_{k=n-n_0}^{n+n_0} x[k]$$

- change of variables: $k'=-n+k$

$$k'=k-n$$

new limits on sum:

$$k\Big]_{n-n_0}^{n+n_0} \Rightarrow k'=\Big]_{n-n_0-n=-n_0}^{n+n_0-n=n_0}$$

- **Substitute:** $k=k'+n$

$$y[n]=\sum_{k'=-n_0}^{n_0} x[n+k']$$

This is a non-causal difference eqn.

You could do another change of variables $k=-k'$ to obtain $$y[n]=\sum_{k=-n_0}^{n_0} x[n-k]$$

• **Is System TI?** First, note: since n_0 is used in system equation, use ℓ for time-shift in input signal:

$$x[n-\ell] \longrightarrow \boxed{S} \longrightarrow z[n]$$

$$\boxed{\text{Is } z[n] = y[n-\ell]?}$$

$$z[n] = \sum_{k=n-n_0}^{n+n_0} x[k-\ell]$$

change of variables: $k' = k - \ell$

new limits:

$$k\Big]_{n-n_0}^{n+n_0} \Rightarrow k'\Big]_{n-n_0-\ell}^{n+n_0-\ell}$$

substitute: $\boxed{k = k' + \ell}$

$$z[n] = \sum_{k'=n-\ell-n_0}^{n-\ell+n_0} x[k']$$

Recall:

$$y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] \implies \text{Thus: } y[n-\ell] = \sum_{k=n-\ell-n_0}^{n-\ell+n_0} x[k]$$

Thus: $z[n] = y[n-\ell] \Rightarrow$ system is TI

Part (c): Is system stable? Yes

- as long as $x[n] < B$ for all n, where $B < \infty$
- Then the "worst case" or largest value $y[n]$ can be if $x[k] = B$ within the window $n-n_0 \leq k \leq n+n_0$
- Thus, $y[n] < (2n_0+1)B$

$\Rightarrow$ system is stable