Prob. 1.26 Help

Consider two DT periodic signals $\left\{ \begin{array}{ll} x_1[n] & \text{periodic} = N_1 \\ x_2[n] & \text{period} = N_2 \end{array} \right.$

implies: $x_1[n + mN_1] = x_1[n]$
for any n, m integer

$x_2[n + \ell N_2] = x_2[n]$
for any n, ℓ integer

- Consider sum:

$$y[n] = x_1[n] + x_2[n]$$

- Sum is periodic with period $N_1 N_2$

$$y[n + N_1 N_2] = x_1[n + N_2 N_1] + x_2[n + N_1 N_2]$$
$$= x_1[n] + x_2[n]$$
$$= y[n]$$

- repeats more quickly if N_1 and N_2 have common divisors:

- Fundamental period $= \dfrac{N_1 N_2}{\gcd(N_1, N_2)}$

- gcd = greatest common divisor
- this result extends for more than 2 DT periodic signals being summed

- <u>Note</u>: product of 2 sinewaves = sum of 2 sinewaves at sum and difference frequencies

$$\cos(\omega_1 n)\cos(\omega_2 n) = \frac{1}{2}\cos((\omega_1+\omega_2)n) + \frac{1}{2}\cos((\omega_2-\omega_1)n)$$

Prob. 1.36 Help/Observations

- **CT sinewave:** $x(t) = e^{j\omega_o t}$
- **DT sinewave:** $x[n] = e^{j\omega_d n}$
- Sampling CT sinewave every T secs:

$$x[n] = x(t)\Big|_{t=nT} = e^{j\omega_o nT} = e^{j(\omega_o T)n}$$

- So: $\omega_d = \omega_o T$ = frequency of DT sinewave
- From class: $\frac{\omega_d}{2\pi} = \frac{p}{q}$ p, q are integers

for $e^{j\omega_d n}$ to be periodic (frequency/2π must be rational)

- **Substitute:** $\omega_d = \omega_o T$ and $\omega_o = \frac{2\pi}{T_o}$

$$\frac{1}{2\pi}\,\frac{2\pi}{T_0}\,T = \frac{T}{T_0} = \frac{p}{q} = \frac{p'}{q'}$$

$$\boxed{\begin{array}{l} p',\,q' \\ \text{no} \\ \text{common} \\ \text{divisors} \end{array}}$$

Cross-multiplying: $q'T = p'T_0$

- T_0 = period of CT sinewave
- T = time between samples

$$q' = \frac{q}{\gcd(p,q)}$$ is period of DT sinewave

gcd = greatest common divisor

- CONCLUSIONS:
  - must be integer no. of samples over an integer no. of periods of the DT sinewave for the DT sinewave to be periodic

- The q' samples forming one period of the DT sinewave may have come from several (p') periods of the CT sinewave

- For example: $x[n] = e^{j 2\pi \frac{3}{8} n}$

is periodic with period = 8

and those 8 samples (per period) came from 3 successive periods of the CT sinewave that was sampled

Prob. 1.54 Help

Can prove: $\sum_{n=0}^{N-1} \alpha^n = \frac{1-\alpha^N}{1-\alpha}$ by induction

Alternatively, set up 2 eqns in 2 unknowns:

$$y[N-1] = \sum_{n=0}^{N-1} \alpha^n \qquad y[N] = \sum_{n=0}^{N} \alpha^n$$

$$\left.\begin{aligned} y[N] - y[N-1] &= \alpha^N \quad (A) \\ y[N] - \alpha y[N-1] &= 1 \quad (B) \end{aligned}\right\}$$

examine (B) − (A)

(B) − (A): $(1-\alpha)\, y[N-1] = 1 - \alpha^N$

$$y[N-1] = \frac{1-\alpha^N}{1-\alpha} = \sum_{n=0}^{N-1} \alpha^n$$

} will use often in course

(b) if $|\alpha| < 1$, then $\lim_{N \to \infty} \alpha^N = 0$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

(c) Take derivative wrt α on both sides:

$$\sum_{n=0}^{\infty} n \alpha^{n-1} = \frac{1}{(1-\alpha)^2}$$

$$\sum_{n=0}^{\infty} n \alpha^{n} = \frac{\alpha}{(1-\alpha)^2}$$