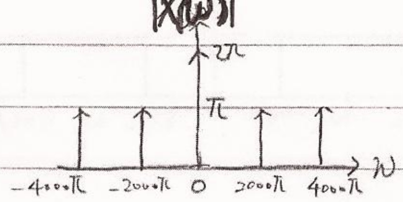
ECE301 Spring 2021
HW #8 Solutions

Text Prob. 7.3.

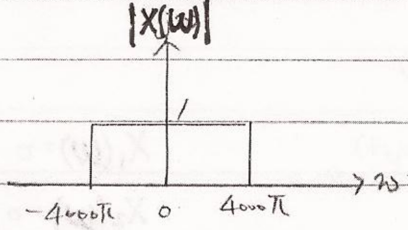
(a) $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$



$BW = 4000\pi$ $\qquad \omega = 2\pi f,\ f = \frac{\omega}{2\pi}$

$\therefore$ Nyquist sampling rate $= 2BW = 8000\pi$ [rad/s]
$= 4000$ [Hz].

(b) $x(t) = \sin(4000\pi)t/(\pi t)$

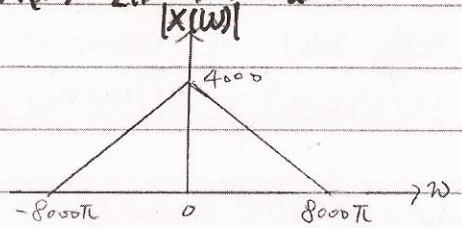


$BW = 4000\pi$

$\therefore$ Nyquist sampling rate $= 2BW = 8000\pi$ [rad/s]
$= 4000$ [Hz]

(c) $x(t) = \left(\sin(4000\pi)t/(\pi t)\right)^2 = x_b^2(t)$

$X(\omega) = \frac{1}{2\pi} X_b(\omega) * X_b(\omega)$



$BW = 8000\pi$

$\therefore$ Nyquist sampling rate $= 2BW = 16000\pi$ [rad/s]
$= 8000$ [Hz]

Text Prob. 7.4. $x(t)$ has Nyquist rate ω_0. $\rightarrow X(\omega) = 0$ for $|\omega| > \omega_0/2$

(a) $y(t) = x(t) + x(t-1) \overset{FT}{\longleftrightarrow} (1 + e^{-j\omega}) X(\omega) = Y(\omega)$

$|Y(\omega)| = |(1 + e^{-j\omega}) X(\omega)| = |1 + e^{-j\omega}||X(\omega)| = 0$ for $|\omega| > \omega_0/2$.

$\therefore$ Nyquist rate of $y(t) = \omega_0$

(b) $y(t) = dx(t)/dt \overset{FT}{\longleftrightarrow} j\omega X(\omega) = Y(\omega)$

$|Y(\omega)| = |j\omega X(\omega)| = |\omega||X(\omega)| = 0$ for $|\omega| > \omega_0/2$

$\therefore$ Nyquist rate of $y(t) = \omega_0$

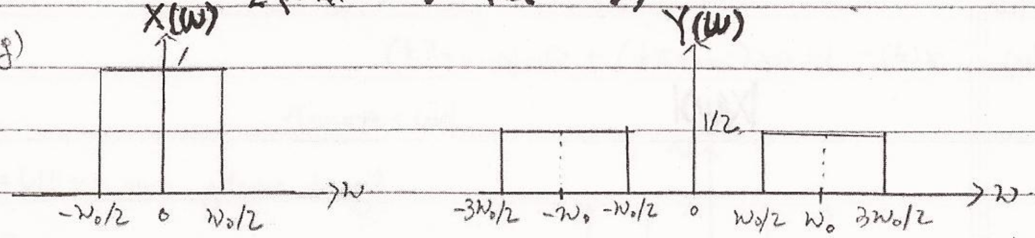
(c) $y(t) = x^2(t) \overset{FT}{\longleftrightarrow} (1/2\pi) X(\omega) * X(\omega) = Y(\omega)$.

We can guarantee that $Y(\omega) = 0$ for $|\omega| > \omega_0$ (Eg) Prob 7.3.(c))

$\therefore$ Nyquist rate of $y(t) = 2\omega_0$.

(d) $x(t)\cos\omega_0 t \xleftrightarrow{FT} \frac{1}{2}\left(X(\omega-\omega_0)+X(\omega+\omega_0)\right)=Y(\omega)$

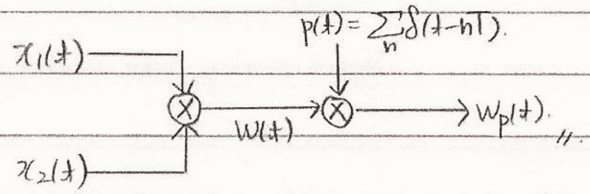
Eg)



$Y(\omega)=0$, for $|\omega|>3\omega_0/2$

$\therefore$ Nyquist rate of $y(t)=3\omega_0$ //

Text Prob. 7.6.



$X_1(\omega)=0$ for $|\omega|\geq\omega_1$

$X_2(\omega)=0$ for $|\omega|\geq\omega_2$

$W(\omega)=\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$

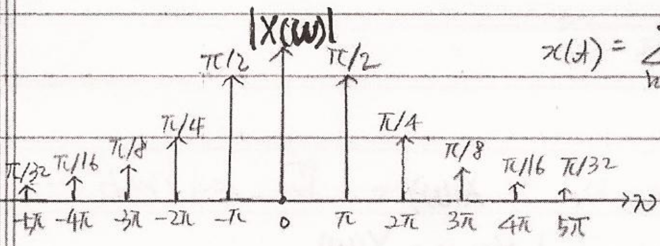
$\rightarrow W(\omega)=0$ for $|\omega|\geq\omega_1+\omega_2$

Nyquist sampling rate (= minimum sampling rate) to reconstruct $w(t)=2(\omega_1+\omega_2)$

$\therefore$ Maximum sampling interval $T=2\pi/2(\omega_1+\omega_2)=\pi/(\omega_1+\omega_2)$ //

Text Prob. 7.8

(a)



$x(t)=\sum_{k=0}^{5}(1/2)^k\sin(k\pi t)$

$\therefore$ Nyquist sampling rate (minimum sampling rate) to recover $x(t)=2\cdot5\pi=10\pi$.

$\Rightarrow$ Maximum sampling period $T_s=2\pi/10\pi=0.2$.

Here, the sampling period must be less than 0.2

If $T=0.2$, frequency component at $-5\pi k, 5\pi k, k=1,\ldots$ will be aliased //

(b) For $k=5$, $X(\omega)=(1/2)^5 j\pi\left(\delta(\omega+5\pi)-\delta(\omega-5\pi)\right)$. If we sample $x(t)$ with $T=2$, this component will be nulled.

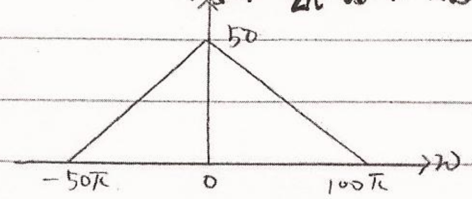
$\therefore x_r(t)=\sum_{k=0}^{4}(1/2)^k\sin(k\pi t) \xrightarrow{F.S.} x_r(t)=\sum_{k=-4}^{4}a_k e^{j(k\pi/t)}$, $a_k=\begin{cases}-j(1/2)^{k+1}, & 1\leq k\leq 4\\ j(1/2)^{-k+1}, & -4\leq k\leq -1\\ 0, & k=0.\end{cases}$

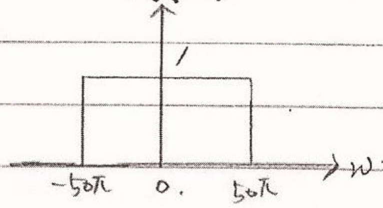
(by LPF w/ cut-off freq $=5\pi$, gain $=0.2$

Text Prob. 7.9.

$x(t) = \left(\dfrac{\sin 50\pi t}{\pi t}\right)^2 = x_0^2(t)$, $\quad x_0(t) = \dfrac{\sin 50\pi t}{\pi t}$

$X(\omega) = \frac{1}{2\pi} X_0(\omega) * X_0(\omega)$ $\qquad$ $X_0(\omega)$





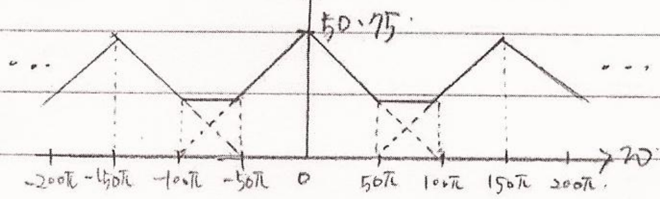
Recall) $x(t)\sum_n \delta(t-nT_s) \xleftrightarrow{\mathcal{F}} (1/T_s)\sum_k X(\omega - k\omega_s)$

Here, $g(t) = x(t)\sum_n \delta(t-nT_s) = x(t)\sum_n \delta(t-n/75)$

where $\omega_s = 150\pi$, $T_s = 2\pi/\omega_s = 1/75$.

$\therefore G(j\omega) = \mathcal{F}\{g(t)\} = 75\sum_{k=-\infty}^{\infty} X(j(\omega - 150\pi k))$.

$G(\omega)$



Therefore, ω_0 satisfying $G(\omega) = 75X(j\omega)$ for $|\omega| \le \omega_0$ is 50π //.

Text Prob. 7.12

$x_d[n] \xleftrightarrow{\mathcal{F}} X_d(e^{j\Omega}) = 0 \quad$ for $3\pi/4 \le |\Omega| \le \pi$.

$x_c(t) = T\sum_n x_d[n] \dfrac{\sin \frac{\pi}{T}(t-nT)}{\pi(t-nT)} \xleftrightarrow{\mathcal{F}} X_c(\omega)$, $\quad T = 10^{-3}$

Note) $\Omega = \omega T$, where Ω = discrete frequency and ω = continuous frequency

Here, for $\Omega = 3\pi/4$, we have $\omega = \dfrac{3\pi}{4}\cdot\dfrac{1}{T} = \dfrac{3\pi}{4}\cdot 1000 = 750\pi$ //

for $\Omega = \pi$, we have $\omega = \pi/T = \pi\cdot 1000 = 1000\pi$

Therefore, $X_c(j\omega) = 0$ for $750\pi \le |\omega| \le 1000\pi$ //