ECE 301, Spring 2024

HW #2 Solution

※ For the notation simplicity, $x[n] \longrightarrow \boxed{S} \longrightarrow y[n]$ ≐ $x[n] \longrightarrow y[n]$ from this page.

ECE301 **1.**

Text Prob 1.18

(a) $x_1[n] \rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$x_2[n] \rightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$.

Let $x[n]$ be the linear compination of $x_1[n]$ and $x_2[n]$.

$$x[n] = ax_1[n] + bx_2[n] \longrightarrow y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] = \sum_{k=n-n_0}^{n+n_0} (ax_1[k] + bx_2[k])$$

$$= a\sum_{k=n-n_0}^{n+n_0} x_1[k] + b\sum_{k=n-n_0}^{n+n_0} x_2[k]$$

$$= ay_1[n] + by_2[n]$$

∴ The system is linear.

(b) $x_1[n] \longrightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$$\underset{\uparrow}{x_2[n]} = x_1[n-n_1] \longrightarrow y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k] = \sum_{k=n-n_0}^{n+n_0} x_1[k-n_1]$$

"shifting $x_1[n]$ in time"

$$= \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k] \quad ''$$

Also, $y_1[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x_1[k]$.

$\Rightarrow y_2[n] = y_1[n-n_1]$.

∴ The system is time-invariant.

(c) $x[n] < B \rightarrow y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] = \underbrace{x[n-n_0] + \dots +}_{\#\text{ of terms} = n_0} x[n] + \underbrace{\dots x[n+n_0]}_{\#\text{ of terms} = n_0}$

$< (2n_0+1)B$.

∴ $C = (2n_0+1)B$.

Text Prob 1.20

$x(t) = e^{j2t} \xrightarrow{S} y(t) = e^{j3t}$

$x(t) = e^{-j2t} \xrightarrow{S} y(t) = e^{-j3t}$

(a) $x_1(t) = \cos(2t)$, $y_1(t) = ?$

Now we can write $x_1(t) = \cos(2t) = \dfrac{e^{j2t} + e^{-j2t}}{2}$

And since Sys. is linear, then the output due to $x_1(t)$:

$x_1(t) = \frac{1}{2} e^{j2t} + \frac{1}{2} e^{-j2t} \xrightarrow{S} y_1(t) = \frac{1}{2} e^{j3t} + \frac{1}{2} e^{-j3t}$

$\Rightarrow y_1(t) = \dfrac{e^{j3t} + e^{-j3t}}{2} = \underline{\underline{\cos(3t)}}$

(b) $x_2(t) = \cos\left(2(t-\frac{1}{2})\right) = \cos(2t-1)$

$x_2(t) = \frac{1}{2} e^{j(2t-1)} + \frac{1}{2} e^{-j(2t-1)} = \frac{1}{2} e^{-j} \cdot e^{j2t} + \frac{1}{2} e^{j} \cdot e^{-j2t}$

Therefore, due to linearity of the system, we have:

$y_2(t) = \frac{1}{2} e^{-j} \cdot e^{j3t} + \frac{1}{2} e^{j} e^{-j3t}$

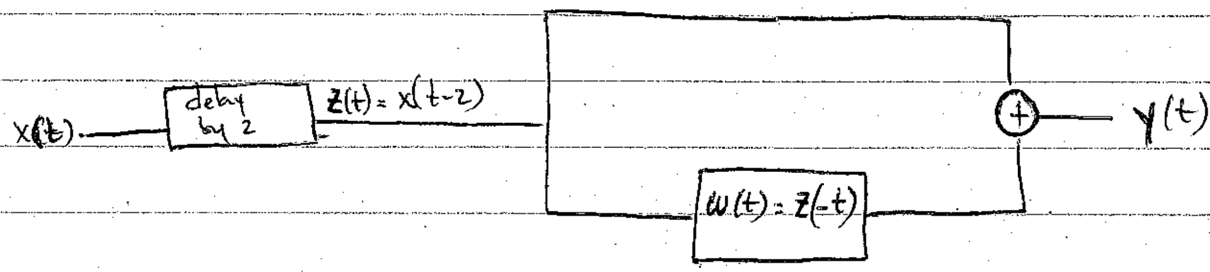
$y_2(t) = \frac{1}{2} e^{j(3t-1)} + \frac{1}{2} e^{-j(3t-1)} = \underline{\underline{\cos(3t-1)}}$

ECE 301, Spring 2014

Hw #2 Solutions

Text Prob 1.27 (a) $y(t) = x(t-2) + x(2-t)$

$y(t)$ can be viewed as following interconnection of sys:



(1) Overall sys $y(t)$ is Linear since each subsys is Linear

(2) $y(t)$ not T.I since $w(t) = z(-t)$ is of the form $y(t) = x(at)$ with $a = -1$, and we know from class that $y(t) = x(at)$ is not T.I

(3) $y(t)$ is not memoryless: $z(t) = x(t-2)$ depends on past

(4) Not Causal since $w(t) = z(-t)$ is not Causal due to properties of $y(t) = x(at)$

(5) $y(t)$ is Stable since both $z(t) = x(t-2)$ and $w(t) = z(-t)$ are stable.

(b) $y(t) = \cos(3t)\, x(t)$

This $y(t)$ is of the form $y(t) = g(t)\, x(t)$. Thus all the properties of $g(t)\, x(t)$ discussed in class hold:

(1) Linear as seen in class.

(2) $y(t)$ is <u>not T.I</u> per $y(t) = g(t)\,x(t)$

(3) <u>Memoryless</u> : $y(t)$ depends only on $x(t)$

(4) <u>Causal</u> : $y(t)$ doesn't depend on future inputs

(5) <u>Stable</u> Since $|g(t)| = |\cos(3t)| \leq 1$

So per properties of $y(t) = g(t)\,x(t)$, the System is <u>stable</u>!

(c) $$y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$$

$$x(t) \longrightarrow \boxed{\int_{-\infty}^{t} x(\tau)\,d\tau} \xrightarrow{z(t)} \boxed{y(t) = z(2t)} \longrightarrow y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$$

(1) $y(t)$ is <u>Linear</u> Since both $z(t)$ (integrator) and $y(t) = z(2t)$ are linear as seen in class.

(2) $y(t)$ is <u>not TI</u> since $y(t) = z(2t)$ of the form $y(t) = x(at)$ is not TI as discussed in class

(3) $y(t)$ is <u>not memoryless</u> per properties of $y(t) = z(2t)$

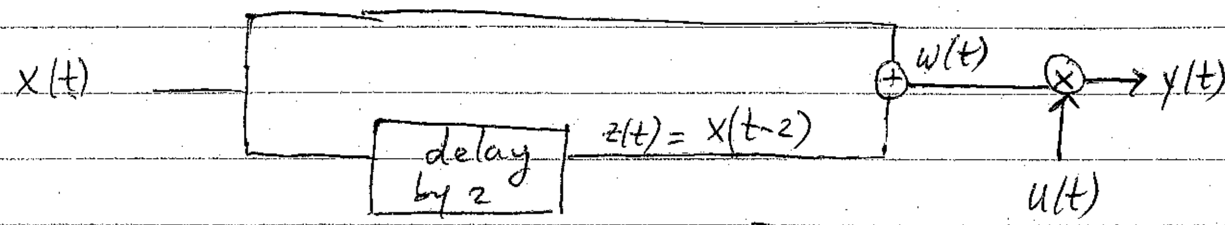
(4) $y(t)$ is <u>not Causal</u> per properties of $y(t) = z(2t)$

(5) $y(t)$ is <u>not stable</u> because the integrator is not stable

i.e. $\lim_{t \to \infty} \int_{-\infty}^{t} x(\tau)\, d\tau \longrightarrow \infty$

d/ $y(t) = \begin{cases} 0 & , t < 0 \\ x(t) + x(t-2) & , t \geq 0 \end{cases}$

Sys. can be viewed as following



i.e $\quad y(t) = w(t)\, u(t) = \left[x(t-2) + x(t)\right] u(t)$

(1) $y(t)$ is <u>Linear</u> since $y(t) = w(t)\,u(t)$ which is of the form $y(t) = x(t)\,g(t)$ is linear and $w(t) = x(t) + x(t-2)$ is itself linear.

(2) $y(t)$ is <u>not T.I</u> per properties of sys of the form $y(t) = g(t)\,x(t)$, where in our case $g(t) = u(t)$.

(3) $y(t)$ is <u>not memoryless</u> since the first subsystem (i.e $z(t) = x(t-2)$) has memory.

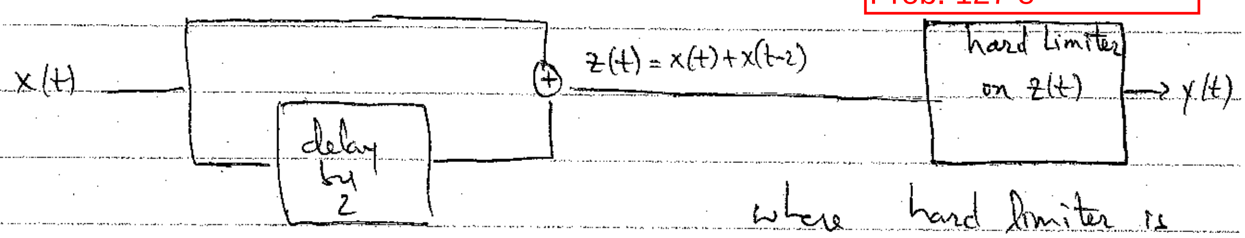
(4) $y(t)$ is <u>Causal</u> since $z(t) = x(t-2)$ is causal and $y(t) = w(t)\,u(t)$ is also causal per properties of $g(t)\,x(t)$

(5) $y(t)$ is <u>Stable</u> since $w(t) = x(t) + x(t-2)$ is stable and $y(t) = w(t)\,u(t)$ is also stable as $|u(t)| = 1$. Thus per properties of $y(t) = x(t)\,g(t)$, then it is stable.

(e) $y(t) = \begin{cases} 0 & , x(t) < 0 \\ x(t) + x(t-2) & , x(t) \geq 0 \end{cases}$

This sys can be viewed as the following interconnection of sys:

This part is not correct: see Hmwk 2 Help for Prob. 127 e



where hard limiter is defined as:

$$y(t) = \begin{cases} \infty & , z(t) > \infty \\ z(t) & , \cancel{x(t-2)} < z(t) < \infty \\ 0 & z(t) < x(t-2) \end{cases}$$

Thus

(1) Sys is <u>not Linear</u> per properties of hard limiter as discussed in class

(2) Sys is <u>T. I</u> Since delay by 2 and hard limiter are both T. I

(3) Sys is <u>not memoryless</u> Since $y(t)$ depends on $x(t-2)$.

(4) Sys is <u>Causal</u> Since $y(t)$ depends only on past $x(t)$

(5) Sys is <u>Stable</u> per properties of hard limiter and time shifting (delay) by 2 doesn't affect the magnitude of the signal.

(f) $y(t) = x\left(\frac{t}{3}\right) = x\left(\frac{1}{3}t\right)$

Sys is of the form $y(t) = x(at)$ with $a = 1/3$

(1) Sys is <u>Linear</u> per properties of $y(t) = x(at)$

(2) Sys is <u>not T.I</u> Same reason as in (1)

(3) Sys is <u>not memoryless</u> since $y(3) = x(1)$

(4) Sys is <u>not Causal</u> Since $y(-3) = x(-1)$

(5) Sys is <u>Stable</u> per properties of $y(t) = x(at)$

(g) $y(t) = \frac{dx(t)}{dt}$

(1) Sys is <u>Linear</u> since $\frac{d}{dt}\left[a x_1(t) + b x_2(t)\right] = a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt}$
$= a\,y_1(t) + b\,y_2(t)$

(2) Sys is <u>T.I</u>

Recall $\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$

so $x_1(t) \longrightarrow y_1(t) = \lim_{\Delta t \to 0} \frac{x_1(t) - x_1(t-\Delta t)}{\Delta t}$

$x_2(t) = x_1(t-t_0) \longrightarrow y_2(t) = \lim_{\Delta t \to 0} \frac{x_1(t-t_0) - x_1(t-t_0-\Delta t)}{\Delta t}$

Now $y_1(t-t_0) = \lim_{\Delta t \to 0} \frac{x_1(t-t_0) - x_1(t-t_0-\Delta t)}{\Delta t}$

so $y_1(t-t_0) = y_2(t)$, thus Sys is <u>T.I</u>

(3) Sys is <u>not memoryless</u> Since $\frac{dx(t)}{dt}$ depends on $x(t-\Delta t)$

(4) Sys is <u>Causal</u> $\frac{dx(t)}{dt}$ depends on $x(t)$ and $x(t-\Delta t)$

per definition of $\frac{dx(t)}{dt} = \lim_{\Delta t \to \infty} \frac{x(t) - x(t-\Delta t)}{\Delta t}$

(5) Sys is <u>not stable</u> Since for $x(t) = u(t)$
then $y(t) = \frac{dx(t)}{dt} = \delta(t)$ and
$|\delta(t)| = \infty$

Text Prob 1.28 (a) $y[n] = x[-n]$

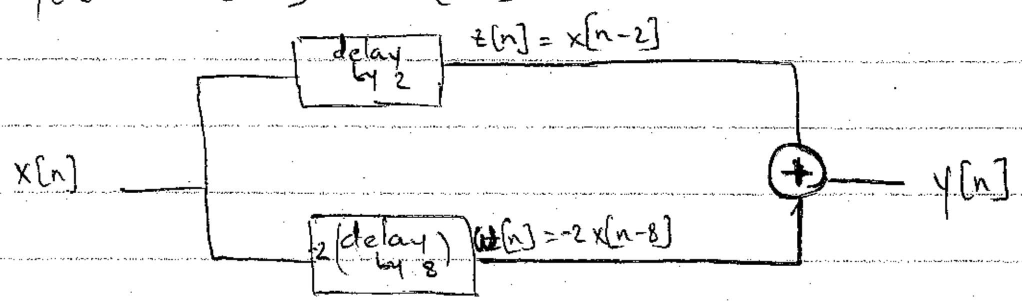
This sys is of the form $y[n] = x[an]$, thus all the properties of $y[n] = x[an]$ hold.

Here $a = -1$.

(1) Sys is <u>Linear</u>
(2) Sys is <u>not T.I</u>
(3) Sys is <u>not memoryless</u>
(4) Sys is <u>not Causal</u>
(5) Sys is <u>Stable</u>

} Per properties of $y[n] = x[an]$ as discussed in class.

b) $y[n] = x[n-2] - 2x[n-8]$

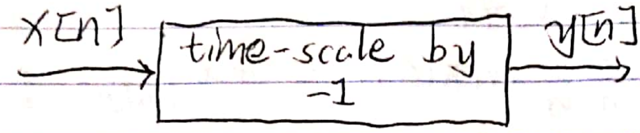


(1) Sys is <u>Linear</u> Since both $z[n]$ and $w[n]$ are Linear

1.28

(a) $y[n] = x[-n]$

System Block Diagram:



(1) System is linear as $y[n] = x[an]$ is linear

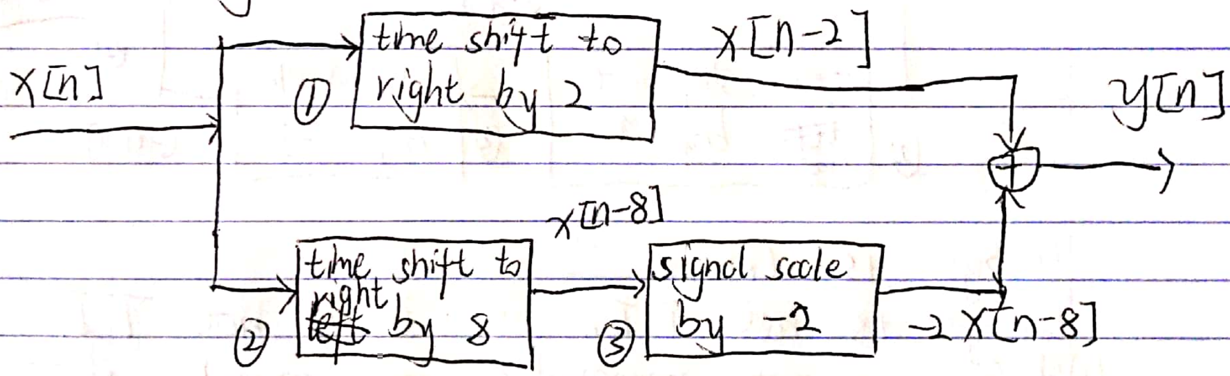
(2) System is not T.I. as $y[n] = x[an]$ is not T.I.

(3)(4) System is not memoryless nor causal: $y[-1] = x[1]$

(5) System is stable

(b) $y[n] = x[n-2] * -2x[n-8]$

Block Diagram:



(1) System is linear

(2) System is T.I.

} Subsystems ①–③ are LTI

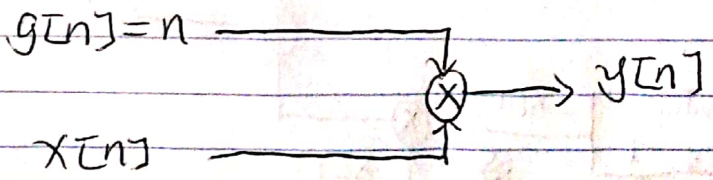
(3) System is causal

(4) System isn't memoryless

(5) System is stable

(c) $y[n] = n\,x[n]$

Block Diagram:

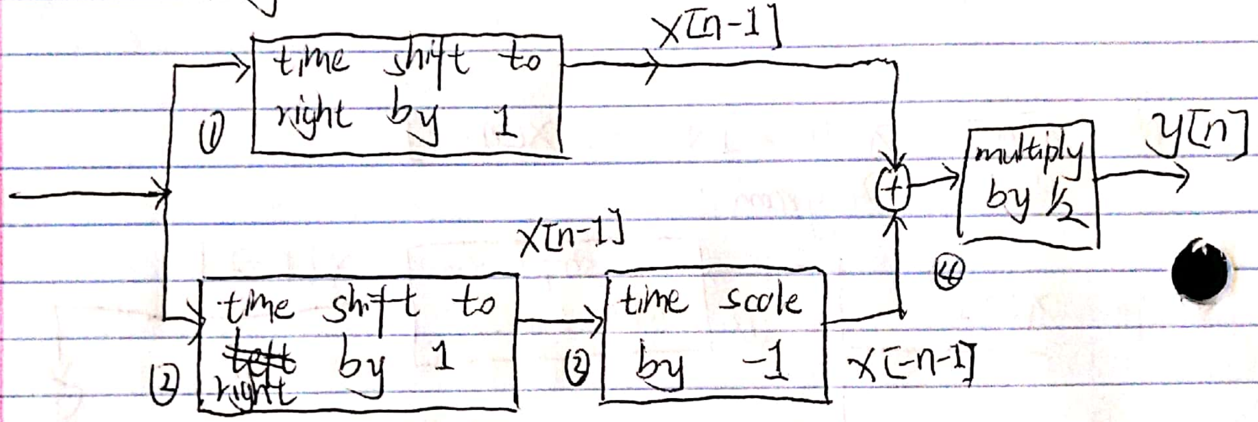


(1) System is linear } a property of $y[n]=g[n]x[n]$
(2) System is not T.I.
(3)(4) System is causal and memoryless
(5) System is not stable:

$$\lim_{n\to+\infty} |y[n]| = \lim_{n\to+\infty} |n\,x[n]| = +\infty \Big|_{x[n]=1}$$

(d) $y[n] = \text{Even}\{x[n-1]\} = \{x[n-1] + x[-n-1]\}/2$

Block Diagram:

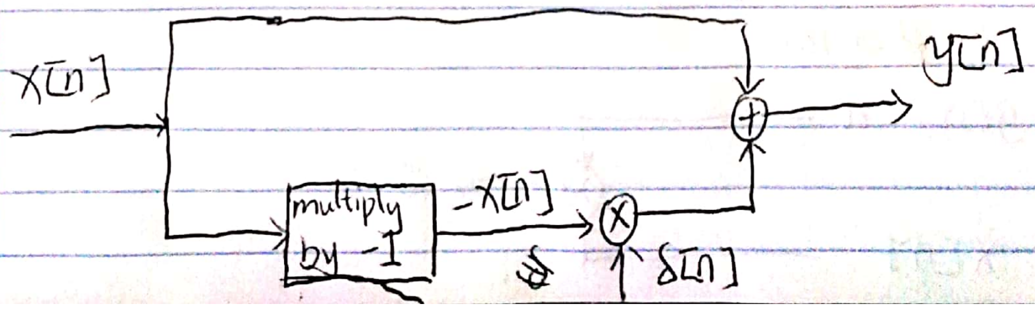


(1) Sys is linear
(2) Sys is not T.I. as ③ is not T.I.
(3)(4) Sys is not memoryless nor causal $y[-2] = \dfrac{x[1]+x[-3]}{2}$
(5) Sys is stable

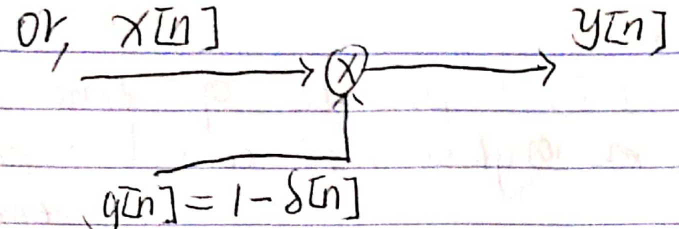
(f) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases} = x[n]\cdot\{1-\delta[n]\} = x[n]\,g[n]$

where $g[n] = 1-\delta[n]$

Block Diagram



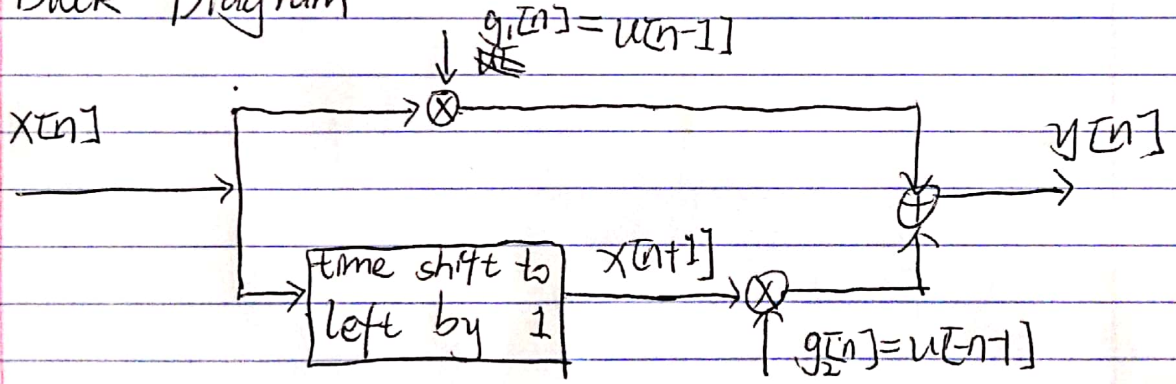
or,



(1) Sys is linear } property of $y[n] = x[n]\,g[n]$
(2) Sys is not T. I.
(3)(4) Sys is memoryless and causal
(5) Sys is stable : $|g[n]| \leq 1 \quad \forall n \in \mathbb{Z}$

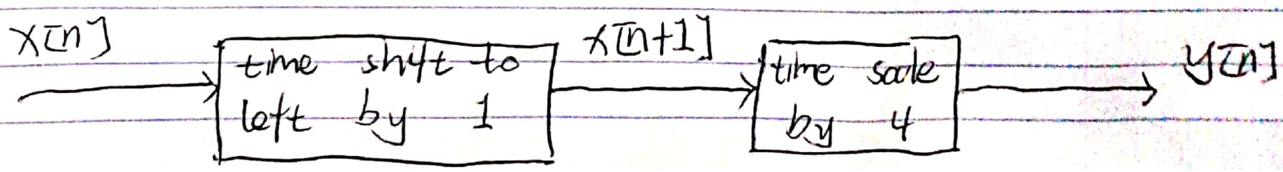
(e) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases} = x[n] \cdot u[n-1] + x[n+1] \cdot u[-n-1]$

Block Diagram



(1) Sys is linear : Both time shift and $y[n] = g[n]x[n]$ are linear
(2) Sys is not T. I. ; $y[n] = x[n] \cdot g[n]$ is not T. I.
(3)(4) Sys is not memoryless nor causal : $y[-1] = x[0]$
(5) Sys is stable

(g) $y[n] = x[4n+1]$
Block Diagram



Scanned with CamScanner

(1) System is linear
(2) System is not T.I. : property of time scale
(3)(4) System is not memoryless nor causal : property of time scale
(5) System is stable