Impulse response for simple first-order Difference Equations

①

$$y[n] = a\,y[n-1] + x[n]$$

- Determine impulse response. (Zero initial conditions)
- When $x[n] = \delta[n]$, $y[n] = h[n]$
- $h[n] = 0$ for $n < 0 \Rightarrow$ causal
- $h[n] = a\,h[n-1] + \delta[n]$ } iterate through recursively

$$\left.\begin{aligned} h[0] &= a\,\cancel{h[-1]}^{\,0} + 1 = 1 = a^0 \\ h[1] &= a\,h[0] = a(1) = a \\ h[2] &= a\,h[1] = a \cdot a = a^2 \\ &\vdots \\ h[n] &= a\,h[n-1] \Rightarrow a^n = a\,a^{n-1} \end{aligned}\right\} h[n] = a^n u[n]$$

- For $a = 1 \Rightarrow y[n] = y[n-1] + x[n]$ ②
- impulse response is unit step

$$h[n] = (1)^n u[n] = u[n]$$

- This is the same impulse response as for the system:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

- Two different realizations of the same system
- And recall (side-note) that the inverse system has the difference equation $y[n] = x[n] - x[n-1]$
- since the impulse response is $h[n] = \delta[n] - \delta[n-1]$

and:

$$u[n] * (\delta[n] - \delta[n-1]) = u[n] - u[n-1] = \delta[n]$$

• Second Example

$$y[n] = a\, y[n-1] + x[n] - a^D x[n-D]$$

- Iterate recursively to find impulse response
- $h[n]=0$ for $n<0$ $\Rightarrow$ causal

$$h[n] = a h[n-1] + \delta[n] - a^D \delta[n-D]$$

$$h[0] = a h[-1] + 1 + 0 = 1$$

$$h[1] = a h[0] + 0 + 0 = a$$

$$\vdots$$

$$h[D-1] = a^{D-1} + 0 + 0 = a^{D-1}$$

$$h[D] = a h[D-1] - a^D (1)$$

$$= a a^{D-1} - a^D = a^D - a^D = 0$$

$$h[D+1] = a h[D] + 0 + 0 = 0$$

$$\Rightarrow h[n] = a^n \{u[n] - u[n-D]\}$$

finite-length geometric sequence

- What is output when $x[n] = b^n\{u[n]-u[n-N]\}$?
- Have multiple options for computing convolution of two finite-length geometric sequences

$$y[n] = \{a^n(u[n]-u[n-D])\} * \{b^n(u[n]-u[n-N])\}$$

when given specific values for a, b, D, N

- <u>Example</u>: $y[n] = \frac{1}{4}y[n-1] + x[n] - \underbrace{\left(\frac{1}{64}\right)}_{=\left(\frac{1}{4}\right)^3}x[n-3]$

Find output when:

$$x[n] = 4 \cdot 4^n\{u[n]-u[n-3]\} = 4^{n+1}\{u[n]-u[n-3]\}$$

↑ recall homogeneity aspect to linearity

- ultimately need to convolve: two length=3 sequences

$$\{\underset{\substack{\uparrow \\ n=0}}{4}, 16, 64\} * \{\underset{\substack{\uparrow \\ n=0}}{1}, \tfrac{1}{4}, \tfrac{1}{16}\}$$

n	0	1	2	3	4	5
• $4\,h[n]$	4	1	1/4	0	0	0
$+\,16\,h[n-1]$	0	16	4	1	0	0
$+\,64\,h[n-2]$	0	0	64	16	4	0
$y[n]$	4	17	68.25	17	4	0

$$y[n] = \{\underset{\substack{\uparrow \\ n=0}}{4}, 17, 68.25, 17, 4\}$$

length = 5
= 3 + 3 − 1