Name: **Solution** — **Final Exam**

ECE301 Signals and Systems — **Wednesday, May 3, 2017**

Cover Sheet

Write your name on this page and every page to be safe.

Test Duration: 120 minutes.

Coverage: Comprehensive

Closed Book. Four two-sided handwritten sheets.

Calculators NOT allowed.

This test contains **four** problems, each with multiple parts.

You have to draw your own plots.

You must show all work for each problem to receive full credit.

Good luck! It was great having you in class this semester!

Have a great Summer!

Problem 1. Clearly label your answer for each part on the following pages.

(a) Determine and plot the Fourier Transform $X_1(\omega)$ of the signal $x_1(t)$ defined below. Indicate which properties/pairs you used to get your answer.

$$x_1(t) = 2j\pi t \left\{ \frac{\sin(5t)}{\pi t} \right\}^2$$

(b) Determine and plot the Fourier Transform $X_2(\omega)$ of the signal $x_2(t)$ defined below.

$$x_2(t) = \frac{1}{2j} \left\{ \frac{\sin(10(t - \frac{\pi}{10}))}{\pi(t - \frac{\pi}{10})} - \frac{\sin(10(t + \frac{\pi}{10}))}{\pi(t + \frac{\pi}{10})} \right\}$$

(c) Given $x_1(t)$ and $x_2(t)$ defined above, the signal $x(t)$ is created as shown below. Determine and plot the Fourier Transform of $x(t)$, $X(\omega)$, of $x(t)$, showing as much detail as possible.

$$x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$$

For EACH of parts (d) thru (g) of this problem, the signal $x(t)$ from part (c) above is input to an LTI system whose impulse response is given. For EACH part, you must do EACH of the following THREE steps. You MUST show all your work.

(i) Plot the magnitude $|H_i(\omega)|$ of the Fourier Transform of the impulse response $h_i(t)$.

(ii) Plot the magnitude $|Y_i(\omega)|$ of the Fourier Transform of the output signal $y_i(t)$.

(iii) Determine a simple, closed-form expression for the time-domain output $y_i(t)$.

(d) $h_1(t) = \dfrac{2\pi}{5} \left\{ \dfrac{\sin(5t)}{\pi t} \dfrac{\sin(15t)}{\pi t} \right\} \cos(30t)$

(e) $h_2(t) = \delta(t) - \dfrac{\sin(20t)}{\pi t}$

(f) $h_3(t) = 2\dfrac{\sin(5t)}{\pi t} \cos(45t)$

(g) $h_4(t) = \dfrac{2\pi}{5} \left\{ \dfrac{\sin(5t)}{\pi t} \dfrac{\sin(15t)}{\pi t} \right\} \cos(60t)$

(h) Determine and plot the magnitude of the Fourier Transform $Z(\omega)$ of the signal $z(t)$ defined below, where $x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$ as defined in part (c). The trig identity $2\cos(\theta)\cos(\phi) = \cos(\theta + \phi) + \cos(\theta - \phi)$ might be useful, but you can solve the problem any way you like.

$$z(t) = 2x(t)\cos(30t)$$

(i) The signal $w(t)$ is the output obtained with $z(t) = 2x(t)\cos(30t)$ from part (h) as the input to the lowpass filter with impulse response defined below. Plot the Fourier Transform $W(\omega)$ of $w(t)$. Is $w(t) = x_1(t)$?

$$h(t) = \frac{\pi}{5} \left\{ \frac{\sin(5t)}{\pi t} \frac{\sin(15t)}{\pi t} \right\}$$

Plot your answers to Problem 1 here. Clearly label each part. Show all work.

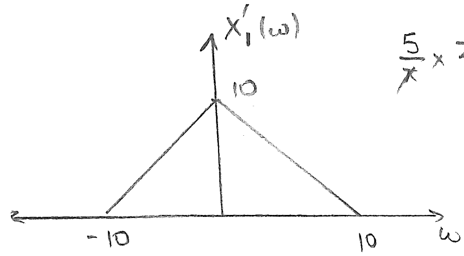
(a) $x_1(t) = 2j\pi t\left\{\frac{\sin(5t)}{\pi t}\right\}^2 = jt\left\{2\pi\frac{\sin 5t}{\pi t}\,\frac{\sin 5t}{\pi t}\right\} = jt\,x_1'(t)$

Let $x_1'(t) = 2\pi\frac{\sin 5t}{\pi t}\,\frac{\sin 5t}{\pi t}$

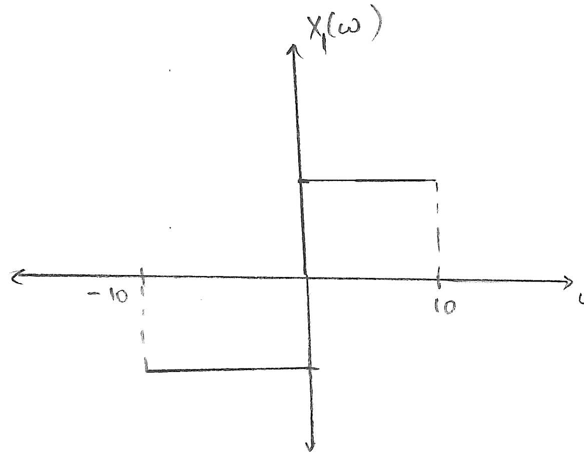
$X_1(\omega) = j\left(j\frac{dX_1'(\omega)}{d\omega}\right)$ $\qquad tx(t) \xleftrightarrow{F} j\frac{d}{d\omega}X(\omega)$

$= -\frac{dX_1'(\omega)}{d\omega}$

$= \begin{cases} -1 & -10<\omega<0 \\ -(-1)=1 & 0<\omega<10 \end{cases}$



$\frac{5}{\cancel{\pi}}\times 2\cancel{\pi} = 10$



(b) $x_2(t) = \frac{1}{2j}\left\{\frac{\sin(10(t-\frac{\pi}{10}))}{\pi(t-\frac{\pi}{10})} - \frac{\sin(10(t+\frac{\pi}{10}))}{\pi(t+\frac{\pi}{10})}\right\}$

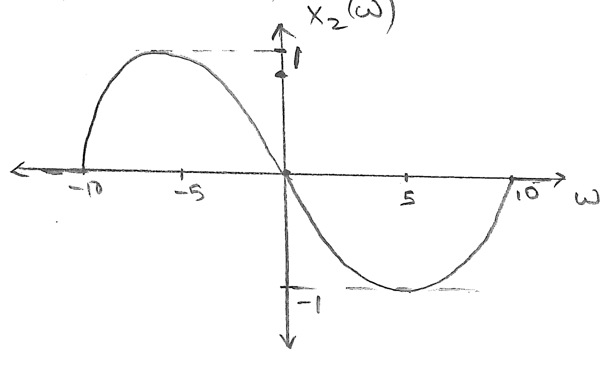
$X_2(\omega) = \frac{1}{2j}\left\{e^{-j\omega\frac{\pi}{10}}\,\mathrm{rect}\left(\frac{\omega}{20}\right) - e^{j\omega\frac{\pi}{10}}\,\mathrm{rect}\left(\frac{\omega}{20}\right)\right\}$

$= -\mathrm{rect}\,\frac{\omega}{20}\,\sin\left(\frac{\pi}{10}\omega\right)$

$\because \frac{\sin Wt}{\pi t} \xleftrightarrow{F} \begin{cases} 1 & |\omega|<W \\ 0 & |\omega|>W \end{cases}$

$x(t-t_0) \xleftrightarrow{F} e^{-j\omega t_0}X(\omega)$

$\sin(\omega_0 t) = \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$

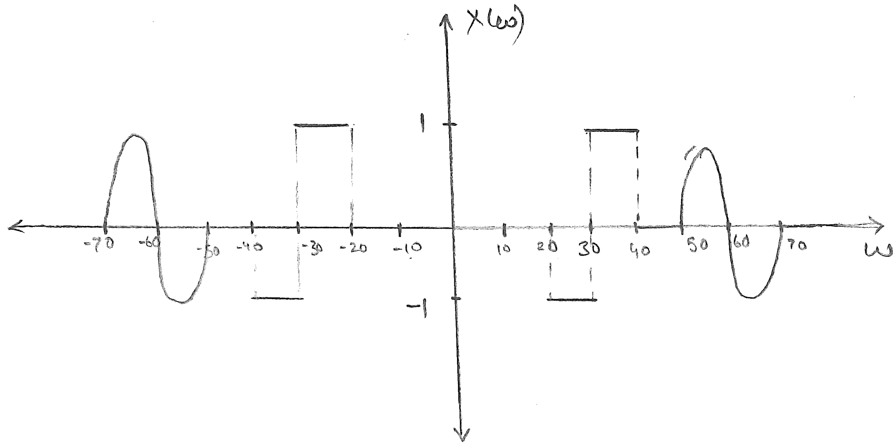


Plot your answers to Problem 1 here. Clearly label each part. Show all work.

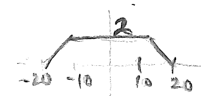
(c) $x(t) = 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$

Since $x(t)\cos(\omega_0 t) \xleftrightarrow{F} \frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0)$

$X(\omega) = X_1(\omega-30) + X_1(\omega+30) + X_2(\omega-60) + X_2(\omega+60)$

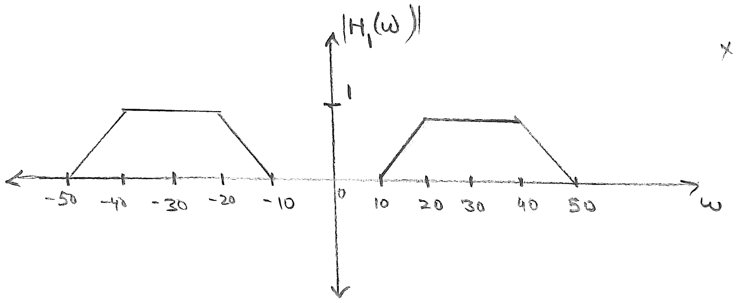


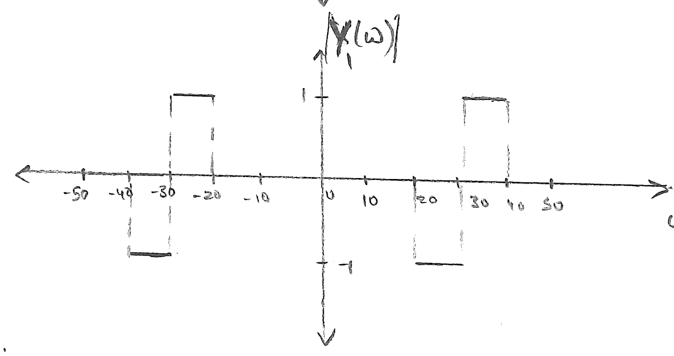
$\frac{2\pi}{5} \times \frac{\omega_1}{\pi} = \frac{5}{\pi} \times \frac{2\pi}{5} = 2$



(d) $h_1(t) = \frac{2\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\,\frac{\sin(15t)}{\pi t}\right\}\cos(30t)$

Using $x(t)\cos(\omega_0 t) \xleftrightarrow{F} \frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0)$





Content at $|\omega| < 10$ & $|\omega| > 50$ is filtered out

~~$y(t)$~~ $Y_1(\omega) = \text{rect}\left(\frac{\omega+30}{20}\right) + \text{rect}\left(\frac{\omega-30}{20}\right)$

$\updownarrow F$

$$\boxed{y_1(t) = \cancel{4}\,j\pi t\left[\frac{\sin(5t)}{\pi t}\right]^2\cos(30t)}$$

$= 2\cancel{0}\,x_1(t)\cos(30t)$

$\because X(\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases} \xleftrightarrow{F} \frac{\sin Wt}{\pi t}$

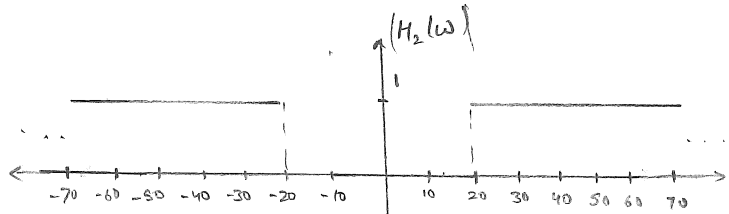
$\frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0) \xleftrightarrow{F} x(t)\cos(\omega_0 t)$

Plot your answers to Problem 1 here. Clearly label each part. Show all work.

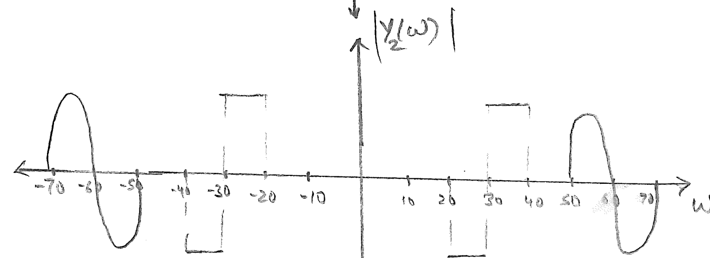
(e)

$$h_2(t) = \delta(t) - \frac{\sin(20t)}{\pi t}$$

$$H_2(\omega) = 1 - \text{rect}\left(\frac{\omega}{40}\right)$$



$$y_2(t) = x(t)$$
$$= 2x_1(t)\cos(30t) + 2x_2(t)\cos(60t)$$
$$= 4j\pi t\left\{\frac{\sin(5t)}{\pi t}\right\}^2 \cos(30t)$$
$$+ \frac{1}{j}\left\{\frac{\sin(10(t-\frac{\pi}{10}))}{\pi(t-\frac{\pi}{10})} - \frac{\sin(10(t+\frac{\pi}{10}))}{\pi(t+\frac{\pi}{10})}\right\}$$

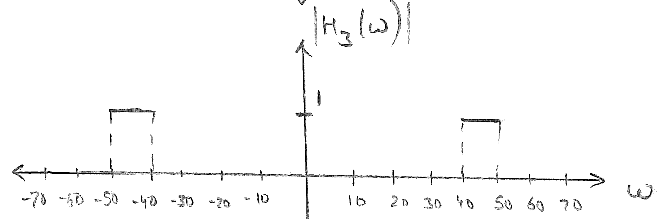


(f)

$$h_3(t) = \frac{2\sin(5t)}{\pi t}\cos(45t)$$

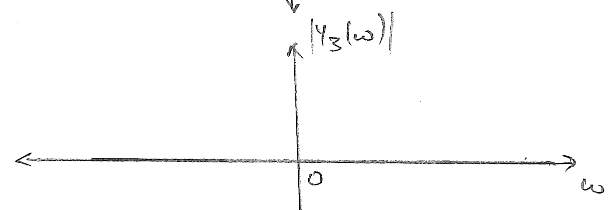
$$H_3(\omega) = \frac{1}{2}\left\{2\,\text{rect}\left(\frac{\omega-45}{10}\right) + 2\,\text{rect}\left(\frac{\omega+45}{10}\right)\right\}$$

$$= \text{rect}\left(\frac{\omega-45}{10}\right) + \text{rect}\left(\frac{\omega+45}{10}\right)$$



$$Y_3(\omega) = 0$$

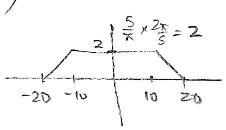
$$\Rightarrow \quad y_3(t) = 0$$

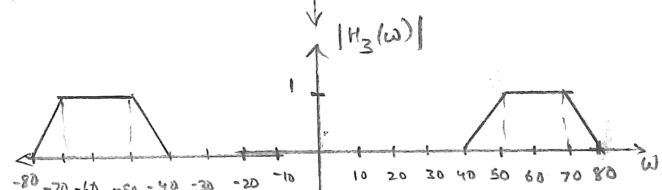


(g)

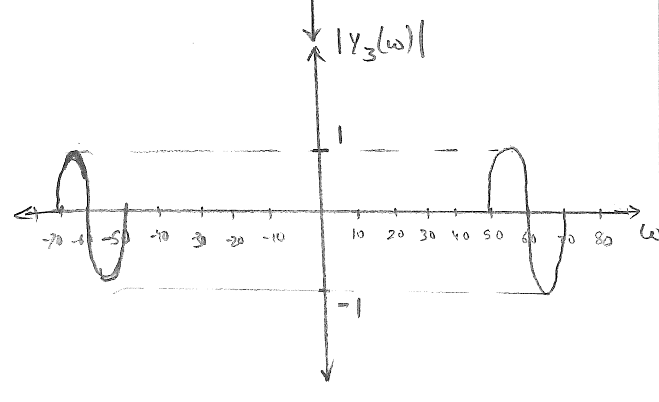
$$h_4(t) = \frac{2\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\;\frac{\sin(15t)}{\pi t}\right\}\cos(60t)$$

$\frac{5}{\pi}\times\frac{2\pi}{5} = 2$





$$y_2(t) = 2x_2(t)\cos(60t)$$
$$= \frac{1}{j}\left\{\frac{\sin(10(t-\frac{\pi}{10}))}{\pi(t-\frac{\pi}{10})} - \frac{\sin(10(t+\frac{\pi}{10}))}{\pi(t+\frac{\pi}{10})}\right\}\cos(60t)$$



Plot your answers to Problem 1 here. Clearly label each part. Show all work.

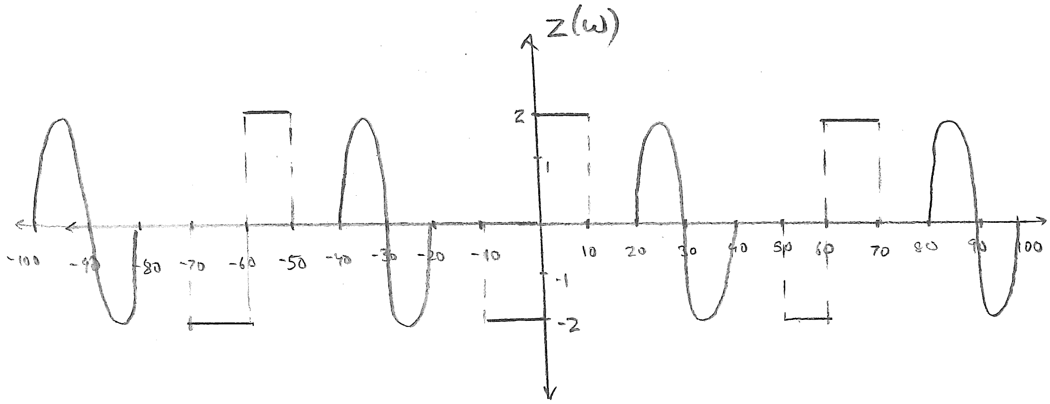
(h) $z(t) = 2x(t)\cos(30t)$

$= 4x_1(t)\cos(30t)\cos(30t) + 4x_2(t)\cos(60t)\cos(30t)$

$= 2x_1(t)\left[\cos(60t) + \cos(0)\right] + 2x_2(t)\left[\cos(30t) + \cos(90t)\right]$

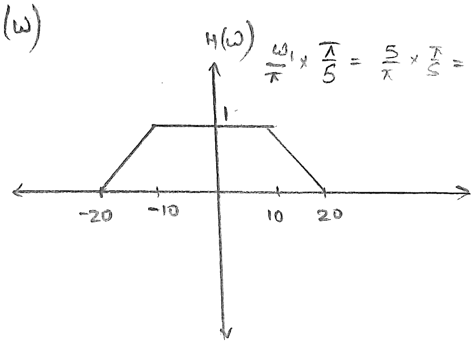
$= 2x_1(t) + 2x_1(t)\cos(60t) + 2x_2(t)\cos(30t) + 2x_2(t)\cos(90t)$

$Z(\omega)$



(i) $h(t) = \frac{\pi}{5}\left\{\frac{\sin(5t)}{\pi t}\ \frac{\sin(15t)}{\pi t}\right\}$

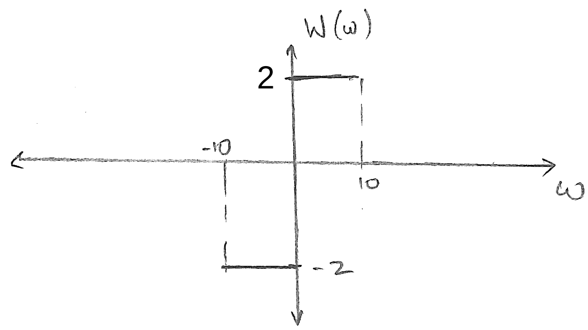
$H(\omega)$



$\boxed{\text{Yes}}$ $W(\omega) = 2X_1(\omega)$

so

$w(t) = 2x_1(t)$



40 pts.

Problem 2.

The rectangular pulse $x_{in}(t) = \{u(t) - u(t-2)\}$ of duration 2 secs is input to the following integrator

$$x_a(t) = 3\int_{t-1}^{t} x_{in}(\tau)d\tau$$

The output $x_a(t)$ is sampled every $T_s = 1/3$ seconds to form $x[n] = x_a(nT_s)$. The sampling rate is $f_s = 3$ samples/sec. Note: Part (b) can be done independently of Part (a). Show work and clearly label and write your final answer in the space provided below or on the next few pages.

(a) Do a stem plot of the DT signal $x[n]$, or you can simply write the numbers that comprise $x[n]$ (indicate with an arrow where the $n = 0$ value is.

(b) Determine a closed-form expression for the DTFT $X(\omega)$. Show work and clearly label and write your final answer in the space provided below or on the next few pages.

(c) The Discrete-Time (DT) signal $x[n]$, created as described above, is input to the DT system described by the difference equation below:

$$y[n] = y[n-1] + x[n] - x[n-4]$$

(i) First, determine and plot the impulse response $h[n]$ for this system. Do the stem-plot for $h[n]$ on the graph provided on the next page.

(ii) Determine and plot the output $y[n]$ by convolving the input $x[n]$ defined above with the impulse response $h[n]$. Show all work in the space provided. Do the stem-plot for $y[n]$ on the graph provided on the page after next.

$3-(t-1)$

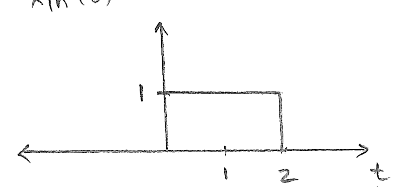
$3-t$

(a) $x[n] = x_a(nT_s)$

$= \{0, 1, 2, 3, 3, 3, 3, 2, 1, 0\}$

$n = \underset{\uparrow}{0} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$

$x_{in}(t)$



$$x_a(t) = 3\int_{t-1}^{t} x_{in}(\tau)\,d\tau = \begin{cases} 0 & t < 0 \\ 3t & 0 \le t < 1 \\ 3 & 1 \le t < 2 \\ 3(3-t) & 2 \le t < 3 \\ 0 & t > 3 \end{cases}$$

(b) $X(\omega) = ??$

Note: $x[n] = (u[n] - u[n-3]) * (u[n] - u[n-6])$

Thus,

$$X(\omega) = e^{-j\frac{(3-1)}{2}\omega}\frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}\, e^{-j\frac{(6-1)}{2}\omega}\frac{\sin\left(\frac{6}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

since the "trapezoidal" sequence starts at n=1, i.e., it's slide over to the right by n0=1, there's an additional linear phase factor in the final answer for the DTFT below

Show your work and plots for Problem 2 here.

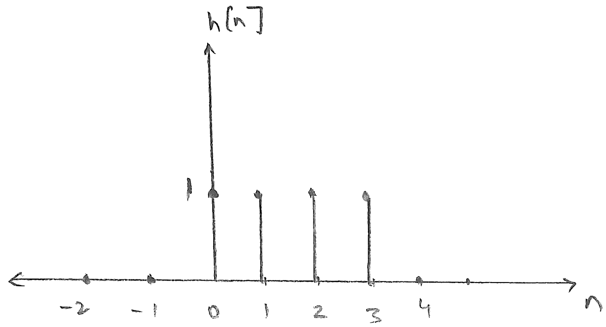
$$X(\omega) = e^{-j\left(\frac{9}{2}\right)\omega} \frac{\sin\left(\frac{3}{2}\omega\right)}{\sin\left(\frac{1}{2}\omega\right)} \frac{\sin(3\omega)}{\sin\left(\frac{1}{2}\omega\right)}$$

(c) $y[n] = y[n-1] + x[n] - x[n-4]$

of form $\left(y[n] = a\,y[n-1] + x[n] - a^D x[n-D]\right)$

(i) $\Rightarrow h[n] = a^n \{u[n] - u[n-D]\}$ where $a = 1$ $D = 4$

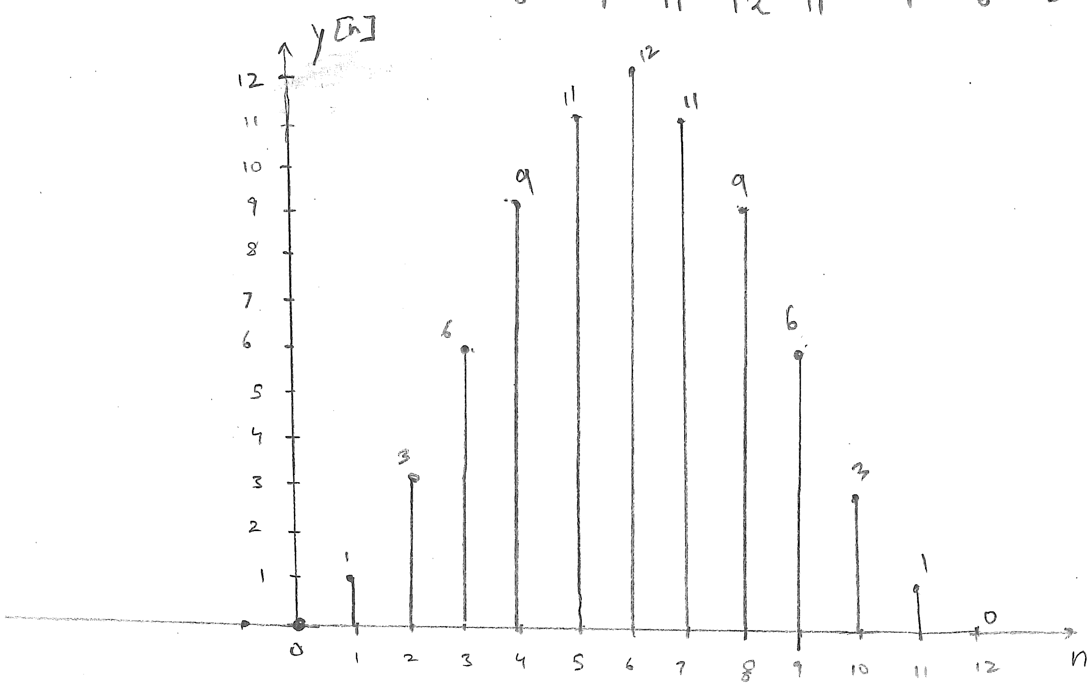
$= u[n] - u[n-4]$



Show your work and plots for Problem 2 here.

(ii) $y[n] = x[n] * h[n]$ $\qquad h[n] = \{1, 1, 1, 1\}$

	n	0	1	2	3	4	5	6	7	8	9	10	11	12
0	$x[0]\,h[n]$	0	0	0	0	0	0	0	0	0	0	0	0	
1	$x[1]\,h[n-1]$		1	1	1	1								
2	$x[2]\,h[n-2]$			2	2	2	2							
3	$x[3]\,h[n-3]$				3	3	3	3						
3	$x[4]\,h[n-4]$					3	3	3	3					
3	$x[5]\,h[n-5]$						3	3	3	3				
3	$x[6]\,h[n-6]$							3	3	3	3			
2	$x[7]\,h[n-7]$								2	2	2	2		
1	$x[8]\,h[n-8]$									1	1	1	1	
0	$\cancel{x[9]\,h[n-9]}$										0	0	0	0
		0	1	3	6	9	11	12	11	9	6	3	1	0



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Problem 3 (a). Consider an analog signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. That is, the Fourier Transform of the analog signal $x_a(t)$ is exactly zero for $|\omega| > 20$ rads/sec. This signal is sampled at a rate $\omega_s = 60$ rads/sec., where $\omega_s = 2\pi/T_s$ such the time between samples is $T_s = \frac{2\pi}{60}$ sec. This yields the discrete-time sequence

$$x[n] = x_a(nT_s) = \left\{\frac{60}{2\pi}\right\}^2 \frac{\sin(\frac{\pi}{4}n)}{\pi n}\frac{\sin(\frac{5\pi}{12}n)}{\pi n} \qquad \text{where:} \quad T_s = \frac{2\pi}{60}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a simple, closed-form expression for the reconstructed signal $x_r(t)$. Show all work.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h(t - nT_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{60} \quad \text{and} \quad h(t) = T_s\frac{\pi}{10}\frac{\sin(10t)}{\pi t}\frac{\sin(30t)}{\pi t}$$

Problem 3 (b). Consider the SAME analog signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. This signal is sampled at the same rate $\omega_s = 60$ rads/sec., where $\omega_s = 2\pi/T_s$ and the time between samples is $T_s = \frac{2\pi}{60}$ sec, but at a different starting point. This yields the Discrete-Time $x[n]$ signal below, where $0 < \epsilon < 1$.

$$x_\epsilon[n] = x_a(nT_s + \epsilon T_s) = \left\{\frac{60}{2\pi}\right\}^2 \frac{\sin(\frac{\pi}{4}(n+\epsilon))}{\pi(n+\epsilon)}\frac{\sin(\frac{5\pi}{12}(n+\epsilon))}{\pi(n+\epsilon)} \qquad \text{where:} \quad T_s = \frac{2\pi}{60}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a simple, closed-form expression for the reconstructed signal $x_r(t)$. Does your final answer depend on the value of ϵ? Explain your answer.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x_\epsilon[n]h(t - (n+\epsilon)T_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{60} \quad \text{and} \quad h(t) = T_s\frac{\pi}{10}\frac{\sin(10t)}{\pi t}\frac{\sin(30t)}{\pi t}$$

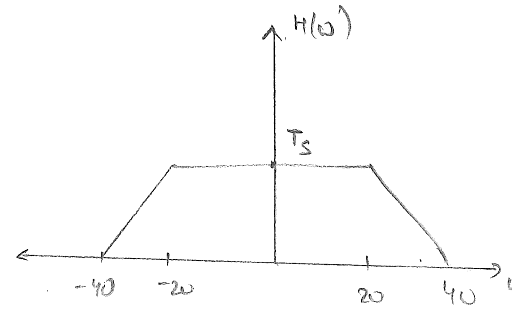
(a) $\omega_M = 20$ rads/s $\qquad \omega_s = 60$ rads/s

$\omega_s > 2\omega_M$ No Aliasing, perfect reconstruction

$$X_r(t) = \sum_{n=-\infty}^{\infty} x[n]\, h(t - nT_s)$$

$$= X_a(t)$$

$$= \frac{\sin\left(\frac{15}{2}t\right)}{\pi t}\frac{\sin\left(\frac{25}{2}t\right)}{\pi t}$$



$\frac{\pi}{4}n =$

ωt

$\omega n T_s = \frac{\pi}{4} n$

$\omega = \frac{\pi}{4} \times \frac{60}{2\pi}$ 15

$nT_s \to t$

$\omega n T_s = \frac{5\pi}{12} n$

$\omega = \frac{5\pi}{12} \times \frac{60}{2\pi}$ 30 25

Show your work for Prob. 3, parts (a)-(b) below.

(b)

$\omega_s > 2\omega_m \Rightarrow$ No Aliasing

$$x_r(t) = \frac{\sin\left(\frac{15}{2}t\right)}{\pi t} \quad \frac{\sin\left(\frac{25}{2}t\right)}{\pi t}$$

No, the answer does not depend on ϵ

$$\frac{\pi}{4}\cancel{\alpha} = \omega_0 \wedge T_s$$

$$\omega_0 = \frac{\cancel{\pi}}{4} \times \frac{60}{2\cancel{\pi}}$$

Problem 3 (c). Consider an analog signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. This signal is sampled at a rate $\omega_s = 40$ rads/sec., where $\omega_s = 2\pi/T_s$ such the time between samples is $T_s = \frac{2\pi}{40}$ sec, yielding the following discrete-time sequence:

$$x[n] = x_a(nT_s) = \left\{\frac{40}{2\pi}\right\}^2 \frac{\sin(\frac{5\pi}{8}n)}{\pi n}\frac{\sin(\frac{3\pi}{8}n)}{\pi n} \qquad \text{where:} \quad T_s = \frac{2\pi}{40}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a closed-form expression for the reconstructed signal $x_r(t)$. Show all work.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h(t - nT_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{40} \quad \text{and} \quad h(t) = T_s\frac{\sin(20t)}{\pi t}$$

Problem 3 (d). Consider the same signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. This signal is sampled at the same rate $\omega_s = 40$ rads/sec., where $\omega_s = 2\pi/T_s$ and the time between samples is $T_s = \frac{2\pi}{40}$ sec, but at a different starting point. This yields the Discrete-Time $x[n]$ signal below, where $0 < \epsilon < 1$.

$$x_\epsilon[n] = x_a(nT_s + \epsilon T_s) = \left\{\frac{40}{2\pi}\right\}^2 \frac{\sin(\frac{3\pi}{8}(n+\epsilon))}{\pi(n+\epsilon)}\frac{\sin(\frac{5\pi}{8}(n+\epsilon))}{\pi(n+\epsilon)} \qquad \text{where:} \quad T_s = \frac{2\pi}{40}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a closed-form expression for the reconstructed signal $x_r(t)$. does your final answer depend on the value of ϵ? Explain your answer.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x_\epsilon[n]h(t - (n+\epsilon)T_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{40} \quad \text{and} \quad h(t) = T_s\frac{\sin(20t)}{\pi t}$$

(c)

$\omega_s = 2\omega_M \Rightarrow$ No Aliasing, perfect reconstruction

$$X_r(t) = x_a(t) = \frac{\sin(\frac{25}{2}t)}{\pi t}\frac{\sin(\frac{15}{2}t)}{\pi t}$$

$$\boxed{\frac{5\pi}{8}n = \omega_1 n T_s = \omega_1 t}$$

$$\Rightarrow \omega_1 = \frac{5\pi}{8} \times \frac{40}{2\pi} = \frac{25}{2}$$

$$\boxed{\frac{3\pi}{8}n = \omega_2 n T_s}$$

$$\Rightarrow \omega_2 = \frac{3\pi}{8} \times \frac{40}{2\pi} = \frac{15}{2}$$

Show your work for Prob. 3, parts (c)-(d) below.

(d)

$x_r(t) = x_a(t)$ perfect reconstruction

$$= \frac{\sin\left(\frac{25}{2}t\right)}{\pi t} \frac{\sin\left(\frac{15}{2}t\right)}{\pi t}$$

No final answer does not depend on ϵ

Problem 3 (e). Consider an analog signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. This signal is sampled at a rate $\omega_s = 30$ rads/sec., where $\omega_s = 2\pi/T_s$ such the time between samples is $T_s = \frac{2\pi}{30}$ sec, yielding the following discrete-time sequence:

$$x[n] = x_a(nT_s) = \left\{\frac{30}{2\pi}\right\}^2 \frac{\sin(\frac{\pi}{2}n)}{\pi n} \frac{\sin(\frac{5\pi}{6}n)}{\pi n} \qquad \text{where:} \quad T_s = \frac{2\pi}{30}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a closed-form expression for the reconstructed signal $x_r(t)$. Show all work.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h(t - nT_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{30} \quad \text{and} \quad h(t) = T_s \frac{\sin(15t)}{\pi t}$$

Problem 3 (f). Consider the SAME analog signal with maximum frequency (bandwidth) $\omega_M = 20$ rads/sec. This signal is sampled at the same rate $\omega_s = 30$ rads/sec., where $\omega_s = 2\pi/T_s$ and the time between samples is $T_s = \frac{2\pi}{30}$ sec, but at a different starting point. This yields the Discrete-Time $x[n]$ signal below, where $0 < \epsilon < 1$.

$$x_\epsilon[n] = x_a(nT_s + \epsilon T_s) = \left\{\frac{30}{2\pi}\right\}^2 \frac{\sin(\frac{\pi}{2}(n+\epsilon))}{\pi(n+\epsilon)} \frac{\sin(\frac{5\pi}{6}(n+\epsilon))}{\pi(n+\epsilon)} \qquad \text{where:} \quad T_s = \frac{2\pi}{30}$$

A reconstructed signal is formed from the samples above according to the formula below. Determine a simple, closed-form expression for the reconstructed signal $x_r(t)$. does your final answer depend on the value of ϵ? Explain your answer.

$$x_r(t) = \sum_{n=-\infty}^{\infty} x_\epsilon[n]h(t - (n+\epsilon)T_s) \qquad \text{where:} \quad T_s = \frac{2\pi}{30} \quad \text{and} \quad h(t) = T_s \frac{\sin(15t)}{\pi t}$$

(e) $\omega_s < 2\omega_M \Rightarrow$ Aliasing, ~~not p~~ imperfect reconstruction

$x_r(t) \neq x_a(t)$

$$= \frac{\sin\left(\frac{25}{2}t\right)}{\pi t} \frac{\sin\left(\frac{15}{2}t\right)}{\pi t}$$

$\omega_s - \omega_M$

10

$\frac{\pi}{2} n = \omega_1 n T_s$

$\omega_1 = \frac{\pi}{2} \times \frac{30}{2\pi}$

$= \frac{15}{2}$

We do not get original signal back because of aliasing. New $\boxed{\omega_M = \omega_s - \omega_M = 10}$

(f)

$\omega_a = \frac{\omega_0}{T_s} = \frac{\pi}{2} \times \frac{30}{}$

Yes, final answer depends on ϵ