EE301 Signals and Systems **Final Exam**
Final Exam **Monday, May 4, 2009**

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **three** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

NOTES:

- You need only plot the magnitude of a DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of T_s is seconds for all parts.
- $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}cos(2\theta)$

Notes:

Great semester! it was a pleasure having each of you in class this semester!
I really enjoyed teaching 301 this semester. **Good luck on the Final!**
Have a great summer!
"

Final Grades: The final grades are due next Monday, May 11.
It takes a while to grade the finals, get them recorded in the database,
rank order every one, determine grade cut-offs, etc.
Thus, the final grades may not be done until Sunday night possibly.
Note: We are not allowed to post grades, not even with a partial SSN.

Problem 1. Consider the discrete-time LTI system described by the following recursive difference equation (*Hint: the impulse response is finite length.*)

$$y[n] = y[n-1] + x[n] - x[n-4] \tag{1}$$

Note that the answer to Part (a) below will be helpful for solving Problem 2.

(a) Determine and plot the impulse response of the system, $h[n]$. There are several ways to find $h[n]$. The simplest way may be by running the input $x[n] = \delta[n]$ through the difference equation above (system is initially at rest - no initial conditions.)

(b) Determine and write a simple, closed-form expression for the frequency response, $H(\omega)$, the DTFT of $h[n]$. Plot magnitude, $|H(\omega)|$, of the frequency response over $-\pi < \omega < \pi$.

  (i) Explicitly list all frequencies within the range $-\pi \leq \omega \leq \pi$ for which $H(\omega) = 0$.

  (ii) Explicitly state the numerical value of $H(0)$.

(c) Determine the time-domain output $y[n]$ obtained when the signal $x[n]$ below is input to the overall system. For this part, plot the magnitude of $Y(\omega)$, the DTFT of $y[n]$.

$$x[n] = \left(1 + \cos\left(\frac{\pi}{2}n\right)\right)^2$$

(d) Determine the time-domain output $y[n]$ obtained when the signal $x[n]$ below is input to the overall system. For this part, plot the magnitude of $Y(\omega)$, the DTFT of $y[n]$.

$$x[n] = 1 + j^n + (-j)^n + (-1)^n$$

(e) Determine the time-domain output $y[n]$ obtained when the signal $x[n]$ below is input to the overall system. For this part plot $y[n]$.

$$x[n] = x_a(nT_s), \qquad x_a(t) = u(t) - u(t-10) \quad \text{and} \quad T_s = 3$$

(f) Determine the output obtained when the signal $x[n]$ below is input to the overall system. For this part, plot the magnitude of $Y(\omega)$, the DTFT of $y[n]$.

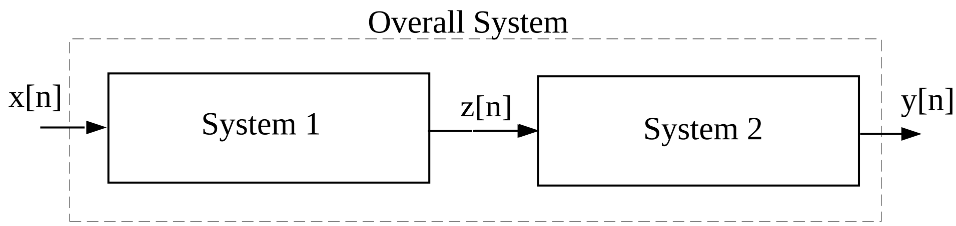
$$x[n] = x_a(nT_s) \quad \text{where} \quad T_s = \frac{2\pi}{20} \quad \text{and} \quad x_a(t) = T_s\frac{\pi}{10}\left\{\frac{\sin(10t)}{\pi t}\right\}^2$$

  (i) For this part, compute the numerical value of $\mathcal{E}_y = \frac{1}{2\pi}\int_{-\pi}^{\pi} |Y(\omega)|^2 d\omega$.

(g) Determine the output obtained when the signal $x[n]$ below is input to the overall system. For this part, plot the magnitude of $Y(\omega)$, the DTFT of $y[n]$.

$$x[n] = x_a(nT_s) \quad \text{where} \quad T_s = 1 \quad \text{and} \quad x_a(t) = \left\{\sum_{k=-\infty}^{\infty} \delta(t - k4)\right\} * \left\{\frac{\sin(\frac{5\pi}{4}t)}{\pi t}\right\}$$

Problem 2. Consider two discrete-time LTI systems in series.



The first system is described by the following difference equation

$$z[n] = z[n-1] + x[n] - x[n-3] \tag{2}$$

The second system is described by the following difference equation:

$$y[n] = y[n-1] + z[n] - z[n-5] \tag{3}$$

(a) Determine and plot the impulse response of the overall system, $h[n]$. You might want to first answer (i) and (ii) below:

  (i) Determine the impulse response of the first system $h_1[n]$

  (ii) Determine the impulse response of the second system $h_2[n]$

(b) Determine and write a simple, closed-form expression for the overall frequency response, $H(\omega)$, the DTFT of $h[n]$.

(c) Plot the magnitude, $|H(\omega)|$, of the overall frequency response over $-\pi < \omega < \pi$.

  (i) Explicitly list all frequencies within the range $0 < \omega < \pi$ for which $H(\omega) = 0$.

  (ii) Explicitly state the numerical value of $H(0)$.

(d) Determine and write an expression for the time-domain output $y[n]$ obtained when the signal $x[n]$ below is input to the overall system

$$x[n] = \left(1 + \cos\left(\frac{2\pi}{5}n\right)\right)^2 + \cos\left(\frac{2\pi}{3}n\right)$$

Problem 3. A discrete-time signal is created by sampling a continuous-time signal as $x[n] = x_a(nT_s)$ where

$$x_a(t) = \cos(6t) + \cos(12t)$$

and the time between samples is given by $T_s = \frac{2\pi}{12}$.

(a) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(b) $x[n]$ is passed through the parallel DT linear system shown below.

(i) The difference equation characterizing the top system is given below. Plot magnitude of the frequency response $H_a(\omega)$ of the top system over $-\pi < \omega < \pi$.

$$y_a[n] = \frac{1}{2}y_a[n-1] + x[n] - 2x[n-1]$$

(ii) The difference equation characterizing the bottom system is given below. Plot magnitude of the frequency response $H_b(\omega)$ of the top system over $-\pi < \omega < \pi$.

$$y_b[n] = -\frac{1}{2}y_b[n-1] + x[n] + 2x[n-1]$$

(iii) Plot the magnitude of the frequency response of the overall system $H(\omega)$ over $-\pi < \omega < \pi$. You must clearly indicate the frequencies for which $H(\omega) = 0$ over $-\pi \leq \omega \leq \pi$. **NOTE:** $|H_a(\omega) + H_b(\omega)| \neq |H_a(\omega)| + |H_b(\omega)|$.

(c) $x[n]$ is passed through the parallel DT linear system drawn below yielding the output $y[n]$. Plot magnitude of the DTFT of $y[n]$ over $-\pi < \omega < \pi$.