EE301 Signals and Systems — **Final Exam**
Final Exam — **Monday, April 28, 2008**

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **five** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

NOTES:

- You need only plot the magnitude of a DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of T_s is seconds for all parts.

Notes:

Great semester! it was a pleasure having each of you in class this semester!
I really enjoyed teaching 301 this semester. **Good luck on the Final!**
Have a great summer!
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Final Grades: The final grades are due next Monday, May 5.
It takes a while to grade the finals, get them recorded in the database,
rank order every one, determine grade cut-offs, etc.
Thus, the final grades may not be done until Sunday night possibly.
Note: We are not allowed to post grades, not even with a partial SSN.

Problem 1. Consider the discrete-time LTI system described by the difference equation

$$y[n] = x[n+3] + 2x[n+2] + 3x[n+1] + 3x[n] + 3x[n-1] + 2x[n-2] + x[n-3] \qquad (1)$$

(a) Plot the impulse response of the system, $h[n]$.

(b) Determine and write a simple, closed-form expression for $H(\omega)$, the DTFT of $h[n]$. *Hint:* Think convolution of two discrete-time rectangles of different widths.

(c) Roughly sketch the magnitude, $|H(\omega)|$, of the frequency response over $-\pi < \omega < \pi$.

(i) Explicitly list all frequencies within the range $0 < \omega < \pi$ for which $H(\omega) = 0$.

(ii) Explicitly state the numerical value of $H(0)$.

(d) Is $H(\omega)$ both real-valued and symmetric? If so, provide an explanation as to why $H(\omega)$ is both real-valued and symmetric.

(e) Determine and write an expression for the time-domain output $y[n]$ obtained when the signal $x[n]$ below is input to the LTI system described by Equation (1).

$$x[n] = \cos\left(\frac{2\pi}{5}n\right) + \cos\left(\frac{2\pi}{3}n\right) + \cos\left(\frac{4\pi}{5}n\right)$$

Problem 2. A discrete-time signal is created by sampling a continuous-time signal as $x[n] = x_a(nT_s)$ where

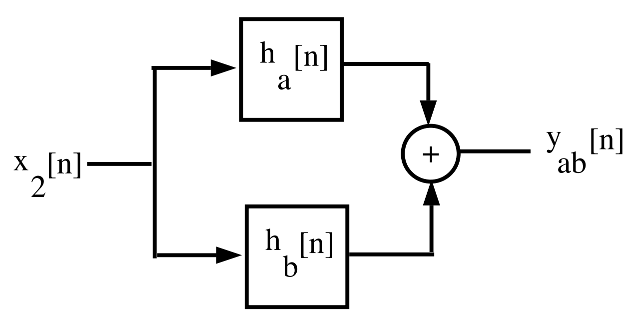
$$x_a(t) = T_s \frac{\sin(Wt)}{\pi t}$$

and the time between samples is given by

$$T_s = \frac{2\pi}{\frac{4}{3}W} = \frac{3\pi}{2W}$$

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through the DT linear system drawn below with impulse response $h_a[n] = 2\left\{\frac{\sin(\frac{\pi}{4}n)}{\pi n}\cos\left(\frac{3\pi}{4}n\right)\right\}$ AND $h_b[n] = 2\left\{\frac{\sin(\frac{\pi}{2}n)}{\pi n}\right\}$ yielding the output denoted $y_{ab}[n]$. Plot magnitude of the DTFT of $y_{ab}[n]$ over $-\pi < \omega < \pi$.



Problem 3.

Consider the continuous-time signal below equal to a sum of four sinewaves.

$$x(t) = \cos(6t) + \cos(12t) + \cos(15t) + \cos(18t) + \cos(21t) + \cos(24t)$$

A discrete-time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \dfrac{2\pi}{24}$.

(a) Plot the magnitude of the DTFT of $x[n]$, $|X(\omega)|$, over $-\pi < \omega < \pi$.

(b) $x[n]$ is passed through a DT linear system described by the difference equation

$$y[n] = \sum_{k=0}^{7} x[n-k]$$

Plot the magnitude of the DTFT of the impulse response of the system, i.e., the frequency response, $|H(\omega)|$, over $-\pi < \omega < \pi$.

(c) Plot magnitude of the DTFT of the output $y[n]$, $|Y(\omega)|$, over $-\pi < \omega < \pi$.

Problem 4.

Consider the continuous-time signal $x_a(t) = \left\{\dfrac{\sin(3t)}{\pi t}\right\}^2 \cos(4t)$. A discrete-time signal is created by sampling $x_a(t)$ according to $x[n] = x(nT_s)$ for $T_s = \dfrac{2\pi}{16}$.

(a) Plot the magnitude of the DTFT of $x[n]$, $|X(\omega)|$, over $-\pi < \omega < \pi$. Show all work.

(b) $x[n]$ is passed through a DT linear system described by the difference equation

$$y[n] = \frac{1}{2}y[n-1] + x[n] - 2x[n-1]$$

Plot $|Y(\omega)|$, the magnitude of the DTFT of the output $y[n]$, over $-\pi < \omega < \pi$.

Problem 5.

Consider the continuous-time signal $x(t) = x_a(t) * \left\{\dfrac{\sin(2t)}{\pi t}\right\}$, where $*$ denotes convolution and $x_a(t)$ is the periodic train of Dirac delta functions described below.

$$x_a(t) = \sum_{k=-\infty}^{\infty} \delta(t - k4\pi)$$

A discrete-time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \dfrac{2\pi}{4}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through a DT linear system with impulse response $h[n] = 2\dfrac{\sin(\frac{\pi}{4}n)}{\pi n}\cos\left(\dfrac{5\pi}{8}n\right)$ yielding the output $y[n]$. Plot magnitude of the DTFT of $y[n]$ over $-\pi < \omega < \pi$.