EE301 Signals and Systems — **Final Exam**
Final Exam — **Monday, April 30, 2007**

Cover Sheet

Test Duration: 120 minutes.
Coverage: Comprehensive.
Open Book but Closed Notes.
Calculators NOT allowed.
This test contains **five** problems.
All work should be done in the blue books provided.
Do **not** return this test sheet, just return the blue books.

NOTES:

- You need only plot the magnitude of a DTFT over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.
- The unit of T_s is seconds for all parts.

Notes:

Great semester! it was a pleasure having each of you in class this semester!
I really enjoyed teaching 301 this semester. **Good luck on the Final!**
Have a great summer!
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Final Grades: The final grades are due next Monday, May 8.
It takes a while to grade the finals, get them recorded in the database,
rank order every one, determine grade cut-offs, etc.
Thus, the final grades will not be done until Sunday night at the earliest.
Note: We are not allowed to post grades, not even with a partial SSN.

Problem 1. Consider the discrete-time LTI system described by the difference equation

$$y[n] = x[n+2] + 2x[n+1] + 3x[n] + 2x[n-1] + x[n-2] \tag{1}$$

(a) Plot the magnitude $|H(e^{j\omega})|$ and phase $\angle H(e^{j\omega})$ (two separate plots) of the frequency response of this system (equal to the DTFT $H(e^{j\omega})$ of the impulse response $h[n]$) as a function of frequency over $-\pi < \omega < \pi$. Show as much detail as possible.

(b) Let $x[n] = x_a(nT_s)$ where $T_s = 0.5$ seconds and $x_a(t) = \cos(2\pi t)\{u(t) - u(t - 0.75)\}$, which is the product of a sinewave with a frequency of 1 Hertz and a rectangular pulse of duration three-fourths of a second. Do the following:

(i) Plot the magnitude $|X(e^{j\omega})|$ of the DTFT of the resulting sampled signal $x[n]$.

(ii) Plot the time-domain output $y[n]$ obtained when $x[n]$ described above is input to the LTI system described by Equation (1).

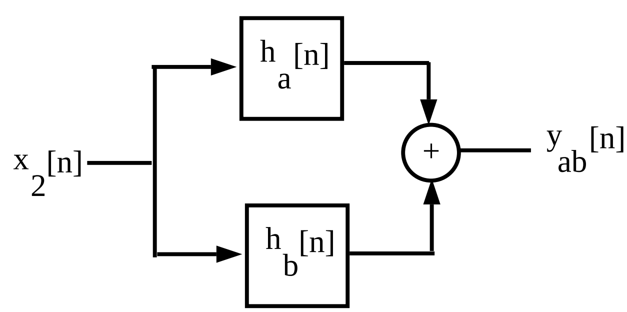
(iii) Plot the magnitude $|Y(e^{j\omega})|$ of the DTFT of the output signal $y[n]$.

(iv) Determine the numerical value of $\sum_{n=-\infty}^{\infty} y^2[n]$. Show all work.

Problem 2. A discrete-time signal is created by sampling a continuous-time signal as $x[n] = x_a(nT_s)$ where $T_s = \frac{2\pi}{6}$ and $x_a(t) = T_s\left\{\frac{\sin(t)}{\pi t} + \frac{\sin(3t)}{\pi t}\right\}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through the DT linear system drawn below with impulse response $h_a[n] = 2(-1)^n\left\{\frac{\sin(\frac{2\pi}{3}n)}{\pi n}\right\}$ AND $h_b[n] = \left\{\frac{\sin(\frac{\pi}{3}n)}{\pi n}\right\}$ yielding the output denoted $y_{ab}[n]$. Plot magnitude of the DTFT of $y_{ab}[n]$ over $-\pi < \omega < \pi$.



Problem 3.

Let $x[n] = x_a(nT_s)$ where $T_s = \dfrac{2\pi}{12}$ and $x_a(t) = 2T_s \left\{ \dfrac{\sin(t)}{\pi t} + \cos(5t) \right\}^2$.

(a) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(b) $x[n]$ is passed through a DT linear system with impulse response $h_a[n] = (-1)^n \left\{ \dfrac{\sin(\frac{\pi}{3}n)}{\pi n} \right\}$ yielding the output $y_a[n]$. Plot magnitude of the DTFT of $y_a[n]$.

(c) $x[n]$ is passed through a DT linear system with impulse response $h_b[n] = 2\cos\left(\dfrac{\pi}{2}n\right) \left\{ \dfrac{\sin(\frac{\pi}{6}n)}{\pi n} \right\}$ yielding the output $y_b[n]$. Plot magnitude of the DTFT of $y_b[n]$ over $-\pi < \omega < \pi$.

(d) $x[n]$ is passed through a DT linear system with impulse response $h_c[n] = \dfrac{\sin(\frac{\pi}{3}n)}{\pi n}$ yielding the output $y_c[n]$. Plot magnitude of the DTFT of $y_c[n]$ over $-\pi < \omega < \pi$.

Problem 4.

Consider the continuous-time signal below equal to a sum of four sinewaves.

$$x(t) = \sin(2t) + \sin(4t + 45^\circ) + \sin(6t + 90^\circ) + \sin(8t + 135^\circ)$$

A discrete-time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \dfrac{2\pi}{8}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

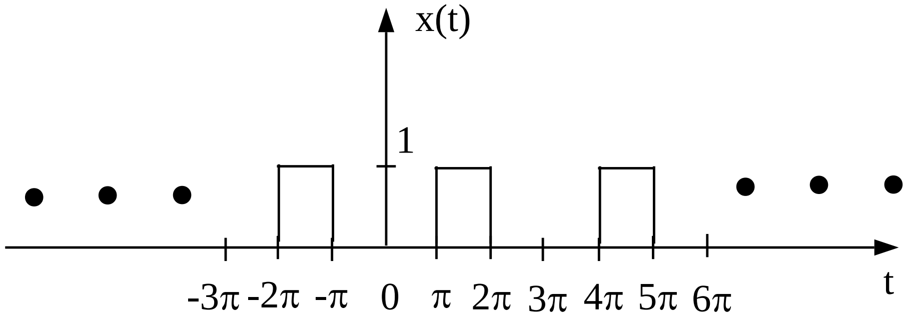
(ii) $x[n]$ is passed through a DT linear system described by the difference equation

$$y[n] = \frac{1}{3}y[n-1] + x[n] - 3x[n-1]$$

Plot magnitude of the DTFT of the output $y[n]$ over $-\pi < \omega < \pi$.

Problem 5.

Consider the continuous-time signal $x(t) = x_a(t) * \left\{ \frac{\sin(2.5t)}{\pi t} \right\}$, where $*$ denotes convolution and $x_a(t)$ is the periodic signal $x(t)$ below with period $T = 3\pi$.



A discrete-time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ for $T_s = \frac{2\pi}{4}$.

(i) Plot the magnitude of the DTFT of $x[n]$ over $-\pi < \omega < \pi$.

(ii) $x[n]$ is passed through a DT linear system with impulse response $h[n] = 2\frac{\sin(\frac{\pi}{4}n)}{\pi n} \cos\left(\frac{\pi}{2}n\right)$ yielding the output $y[n]$. Plot magnitude of the DTFT of $y[n]$ over $-\pi < \omega < \pi$.