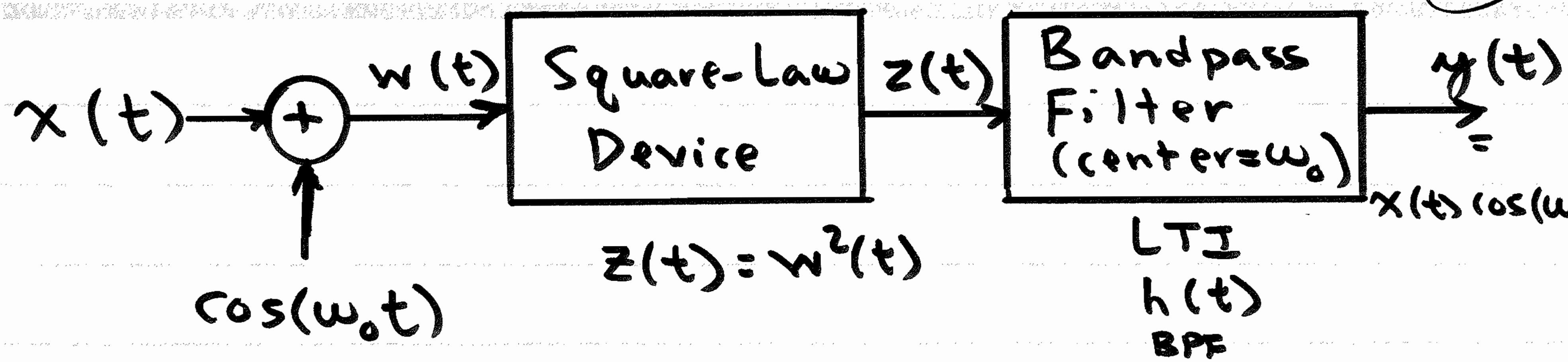
Fourier Transform: Theory and Applications continued: should review 4 handouts posted at the course web site which were covered in class:

- VIP: Proofs of Fourier Transform Properties Plus Examples
- Proofs for Supplemental Properties of the Fourier Transform
- Some Fourier Transform Results involving Two Sinc Functions
- Notes on Response of LTI System to Sinewaves

- How to form the product of a signal with a sinewave? How to form $y(t) = x(t)\cos(\omega_0 t)$?
- done to make a signal radiate from an antenna AND to put signals in different frequency bands so they don't interfere

- Basic trick: put the signal voltage waveform in series with the sinusoidal source, then apply sum of voltages to square-law device
  - square-law device involves diode in combination with voltage-follower

- then filter to isolate the product $x(t)\cos(\omega_0 t)$ while filtering the sienal components near DC (baseband) and at $2\omega_0$ (twice the frequency)



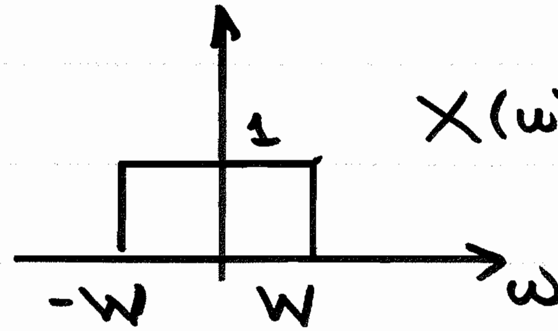
$$z(t) = \left(x(t) + \cos(\omega_0 t)\right)^2$$

$$= x^2(t) + 2\,x(t)\cos(\omega_0 t) + \overbrace{\cos^2(\omega_0 t)}^{\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)}$$

$$= \underbrace{\frac{1}{2} + x^2(t)}_{\substack{\text{spectrally located} \\ \text{in freq. domain} \\ \text{around } \omega = 0 \text{ (baseband)} \\ \Rightarrow \text{filtered out} \\ \Rightarrow \underline{\text{not}} \text{ passed by BPF}}} + \underbrace{2\,x(t)\cos(\omega_0 t)}_{\substack{\text{located near } \omega = \omega_0 \\ \text{in freq. domain} \\ \Rightarrow \text{passed by} \\ \text{BPF}}} + \underbrace{\frac{1}{2}\cos(2\omega_0 t)}_{\substack{\text{located at} \\ \omega = 2\omega_0 \\ \Rightarrow \text{rejected} \\ \text{or } \underline{\text{not}} \text{ passed} \\ \text{by BPF}}}$$

- For simplicity, assume $x(t) = \dfrac{\sin(Wt)}{\pi t}$

$$x(t) = \frac{\sin(Wt)}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow}$$

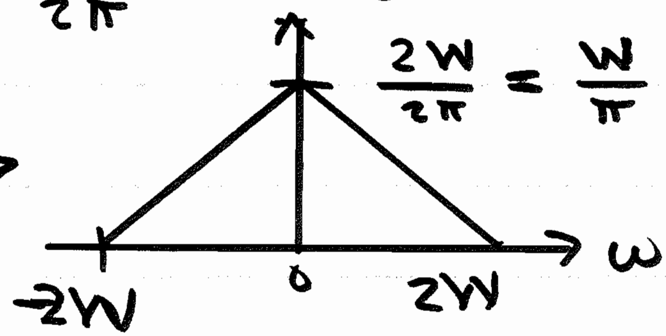


$$X(\omega) = \mathrm{rect}\left(\frac{\omega}{2W}\right)$$

- From multiplication property, we have:

$$x^2(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * X(\omega)$$

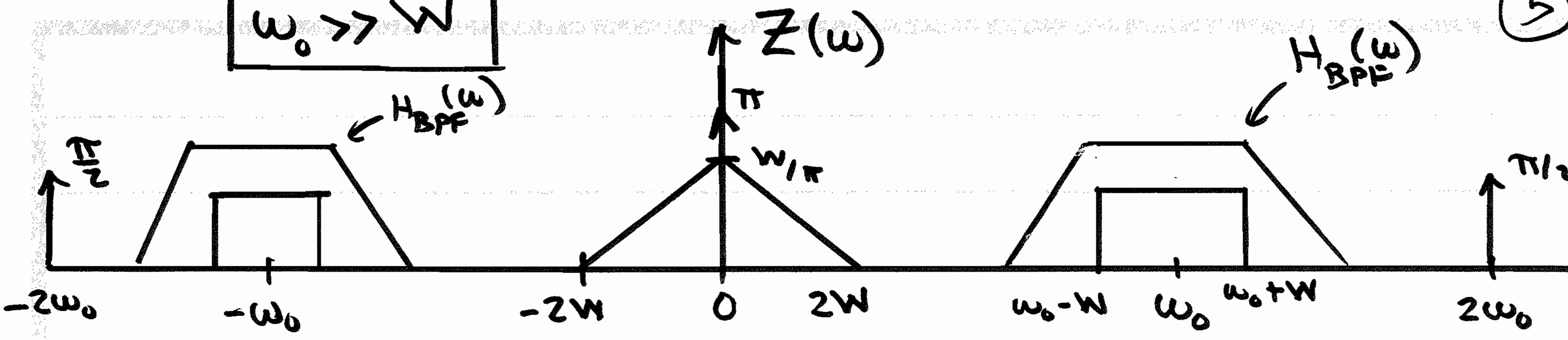
$$\left(\frac{\sin(Wt)}{\pi t}\right)^2 \overset{\mathcal{F}}{\longleftrightarrow}$$



$$Z(\omega) = \frac{1}{2} 2\pi \delta(\omega) + \frac{1}{2\pi} X(\omega) * X(\omega) + X(\omega - \omega_0)$$

$$+ X(\omega + \omega_0) + \frac{1}{2}\pi\delta(\omega - 2\omega_0) + \frac{1}{2}\pi\delta(\omega + 2\omega_0)$$

$$\boxed{\omega_0 >> W}$$



- Have flexibility on BPF design:
  - desire magnitude to be flat over $\omega_0 - W < |\omega| < \omega_0 + W$
  - can roll-off to zero over $2W < |\omega| < \omega_0 - W$
    and $\omega_0 + W < |\omega| < 2\omega_0$

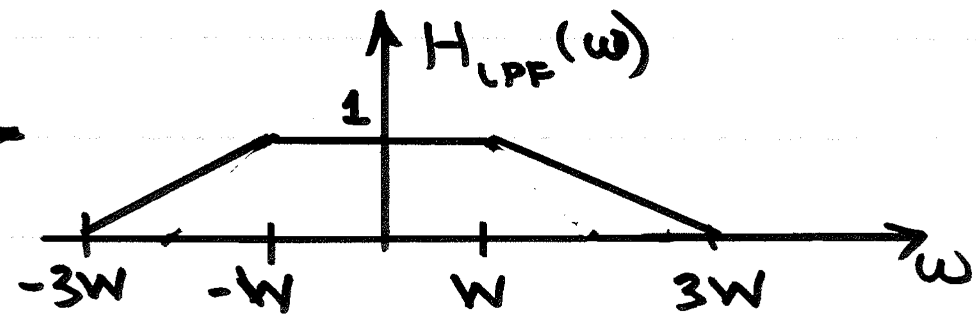
- For example:

$$h_{BPF}(t) = 2\frac{\pi}{W}\frac{\sin(Wt)}{\pi t}\cdot\frac{\sin(2Wt)}{\pi t}\cos(\omega_0 t)$$

$$= 2\, h_{LPF}(t)\cos(\omega_0 t)$$

$$h_{LPF}(t) = \frac{\pi}{W}\,\frac{\sin(Wt)}{\pi t}\,\frac{\sin(2Wt)}{\pi t}$$

$$\overset{\mathcal{F}}{\longleftrightarrow}$$



Then: $h_{BPF}(t) = 2\, h_{LPF}(t)\cos(\omega_0 t)$

$$H_{BPF}(\omega) = H_{LPF}(\omega - \omega_0) + H_{LPF}(\omega + \omega_0)$$

$\Rightarrow$ see plot on previous page

- See Problem 1 of Exam 2 for SP 2010 for Problem on Frequency Division Multiplexing (FDM)