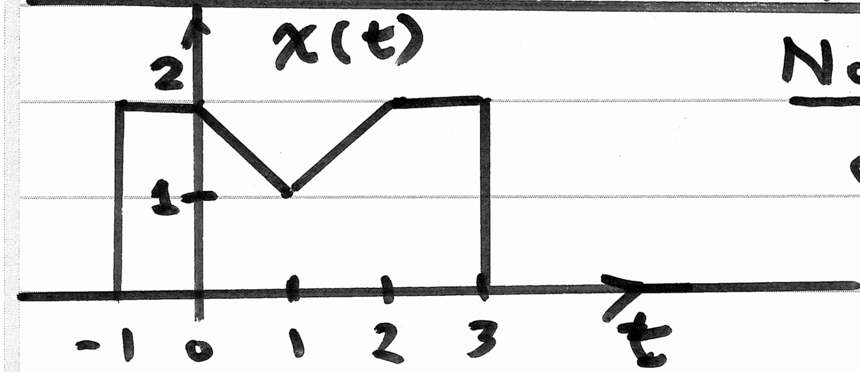
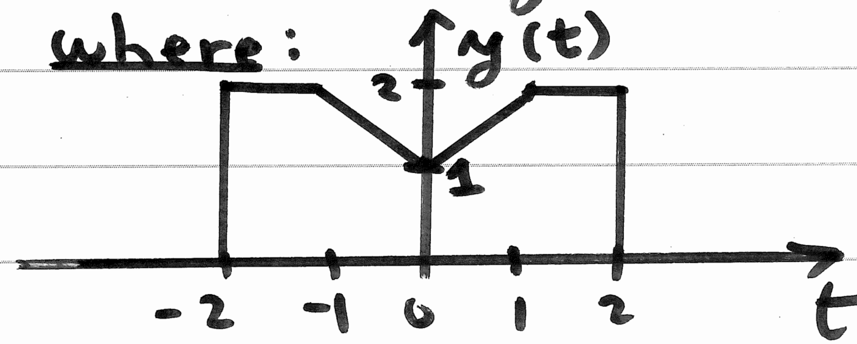
Add'l FT Example. Prob. 4.25 in Text



Note: $y(t) = x(t+1)$

or: $x(t) = y(t-1)$

where:



- $y(t)$ is real-valued and even-symmetric
- Thus: $Y(\omega)$ is real-valued and even-symmetric
- Time-Shift Property dictates:

$$X(\omega) = Y(\omega)\, e^{-j\omega}$$

- Since $Y(\omega)$ is real-valued, this is almost in polar form assuming $Y(\omega) \geq 0$ for all ω

(a) Thus: $\angle X(\omega) = -\omega$

For any value of ω where $Y(\omega) < 0$, add (or subtract) π to phase

(b) Find $X(0) = X(\omega)\Big|_{\omega=0}$

Initial Value Properties dictate:

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \text{area under } x(t)$$

- **Note**: area under $x(t)$ = area under $y(t)$
- **Also**: since $y(t)$ is symmetric, just double the area computed for positive time t
- $$A = 2\left(\int_0^1 (t+1)\,dt + (1)(2)\right)$$

$$= 2\left(\frac{(t+1)^2}{2}\Bigg]_0^1 + 2\right) = 2\left(2 - \frac{1}{2} + 2\right) = 8 - 1 = 7$$

$X(0) = 7$

(c) Find: $\int_{-\infty}^{\infty} X(\omega)\,d\omega \Rightarrow x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega$

Thus: Answer $= 2\pi\, x(t)\big|_{t=0} = 2\pi(2) = 4\pi$

(e) (will do (d) next)

Find: $\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega = 2\pi \int_{-\infty}^{\infty} x^2(t)\,dt$

Note: $x(t)$ and $x(t-t_0)$ have same total energy

Thus: Find energy of $y(t)$

Since: $y(-t) = y(t)$

$$\text{Energy} = 2\int_0^{\infty} y^2(t)\,dt$$

$$= 2\left(\int_0^1 (t+1)^2\,dt + 2^2(1)\right)$$

$$= 8 + 2\left.\frac{(t+1)^3}{3}\right]_0^1 = 8 + \left(\frac{8}{3} - \frac{1}{3}\right)2 = 8 + \frac{14}{3} = \frac{38}{3}$$

Answer: $2\pi\ 38/3$

(d) First, note:

$$y(t) = x(t) * h(t) \xleftrightarrow{\mathcal{F}} Y(\omega) = X(\omega)H(\omega)$$

• **Thus:** $$\int_{-\infty}^{\infty} Y(\omega)\, d\omega = 2\pi\, y(t)\Big|_{t=0} = x(t) * h(t)\Big|_{t=0}$$

• **Also,** recall FT pair:

$$\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{\mathcal{F}} \frac{\sin\left(T\frac{\omega}{2}\right)}{\frac{\omega}{2}}$$

For $T=2$ (two)

$$\text{rect}\left(\frac{t}{2}\right) \xleftrightarrow{\mathcal{F}} \frac{\sin(\omega)}{\omega/2}$$

$T=2$

• Finally, note:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega t}\, d\omega$$

$$y(2) = \int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega 2}\, d\omega\, \frac{1}{2\pi}$$

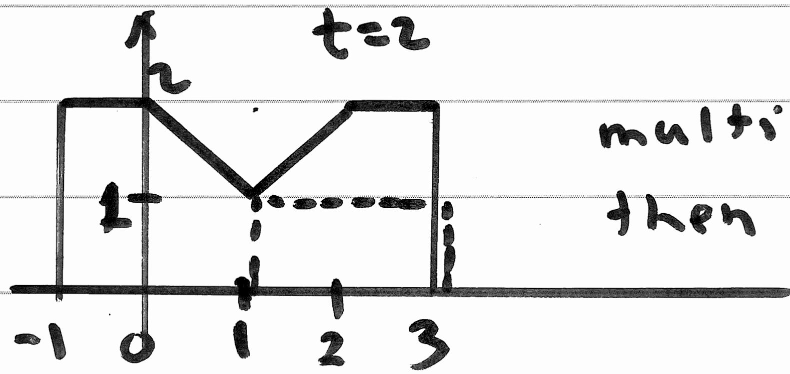
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)H(\omega)\, e^{j2\omega}\, d\omega$$

- Net result of all of this is:

$$\int_{-\infty}^{\infty} X(\omega)\,\frac{\sin(\omega)}{\omega/2}\,e^{j2\omega}\,d\omega =$$

$$2\pi \ast \left\{ \mathrm{rect}\left(\tfrac{t}{2}\right) * x(t) \right\}\Big|_{t=2}$$

times



multiply dashed with solid then find area of / under product

area: $2\times1 + 1\times1 + \frac{1}{2}(1)(1) = 3\frac{1}{2} = \frac{7}{2}$

check: $\int_1^2 t\,dt + (1)(2) = \frac{t^2}{2}\Big]_1^2 + 2 = \left(2-\frac{1}{2}\right)+2 = \frac{7}{2}$ ✓

Final answer: $2\pi\,\frac{7}{2} = 7\pi$