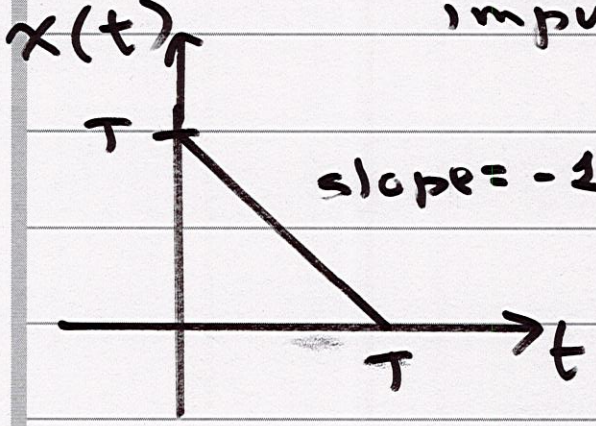
Example. Using "trick"

$$h_d(t) * u(t) = \delta(t)$$

where: $h_d(t) * x(t) = \frac{d}{dt} x(t)$

$h_d(t)$: impulse response of differentiator



$$* \; h(t) = e^{at} u(t) = y(t) = ?$$

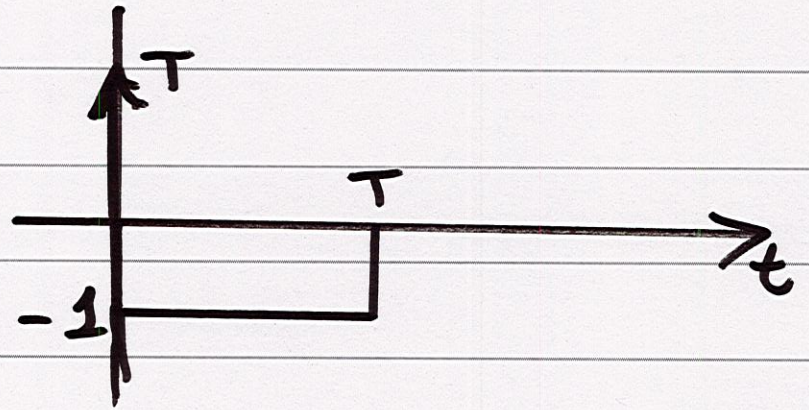
$$y(t) = x(t) * h(t) * \delta(t)$$

$$= x(t) * \delta(t) * h(t)$$

$$= x(t) * h_d(t) * u(t) * h(t)$$

$$= \left\{ \frac{d}{dt} x(t) \right\} * h(t) * u(t)$$

$$\frac{d}{dt}x(t) = T\delta(t) - \{u(t) - u(t-T)\}$$



$$y(t) = \{T\,\delta(t) - (u(t) - u(t-T))\} * e^{at}u(t) * u(t)$$

$$= T\,e^{at}u(t) * u(t) - e^{at}u(t) * u(t) * u(t)$$
$$+ e^{at}u(t) * u(t) * u(t-T)$$

$$e^{at}u(t) * u(t) = \frac{1}{a}e^{at}u(t) - \frac{1}{a}u(t) = z(t)$$

Note: $e^{at}u(t) * e^{at}u(t) = t\,e^{at}u(t)$

$a = 0 \qquad u(t) * u(t) = t\,u(t)$

$$e^{at}u(t) * u(t) * u(t)$$

$$= \left(\frac{1}{a}e^{at}u(t) - \frac{1}{a}u(t)\right) * u(t)$$

$$= \frac{1}{a}\left\{\frac{1}{a}e^{at}u(t) - \frac{1}{a}u(t)\right\} - \frac{1}{a}t\,u(t) = w(t)$$

Final answer:

$$y(t) = T\,z(t) - w(t) + w(t-T)$$

where: w(t) = (1/a) { z(t) - t u(t) }