NAME: **NAME**
EE301 Signals and Systems **In-Class Exam**
Exam 3 **Thursday, Apr. 19, 2018**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 5,7
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
For Problem 4, plot your answers on the graphs provided.

VIP Note Regarding DTFT Plots: *The abscissa in each plot is the frequency axis. For each plot, the abscissa goes from* -2π *to* 2π *with tic marks every* $\pi/8$*. There is a dashed vertical line at* $\omega = -\pi$ *and another dashed vertical line at* $\omega = +\pi$*. You only have to plot any DTFT over* $-\pi < \omega < \pi$*.*

Problem 1 **Short answer questions.**

(a) Briefly write and explain **ONE** of the main advantages of digital over analog, in terms of storage, transmission, and/or processing, in a coherent sentence.

1. Encoding sample amplitudes as 1's and 0's allows for error control/correction coding to automatically correct for bit errors due to noise and imperfections

2. Regeneration for digital communications

3. Flexibility and precision of digital signal processing after A/D conversion

(b) If one samples at a rate ω_s in radians/sec, what analog frequency is the frequency $\frac{\omega_s}{2} + \Delta\omega$ aliased to? Assume $0 < \Delta\omega < \frac{\omega_s}{2}$.

$$\frac{\omega_s}{2} + \Delta\omega - \omega_s = -\frac{\omega_s}{2} + \Delta\omega$$

If the signal is real-valued, e.g.,

$$\cos\left(\left(-\frac{\omega_s}{2} + \Delta\omega\right)t\right) = \cos\left(\left(\frac{\omega_s}{2} - \Delta\omega\right)t\right)$$

Thus: accept either

$$-\frac{\omega_s}{2} + \Delta\omega \quad \text{or} \quad \frac{\omega_s}{2} - \Delta\omega$$

Alternatively: aliasing starts at $\omega_s - \omega_M$

For a sinewave, $\omega_M =$ frequency of sinewave $= \frac{\omega_s}{2} + \Delta\omega$

Thus, aliased to:

$$\omega_s - \left(\frac{\omega_s}{2} + \Delta\omega\right) = \frac{\omega_s}{2} - \Delta\omega$$

Problem 2. Consider the input signal $x_0(t)$ below.

$$x_0(t) = e^{-j32t}+e^{-j28t}+e^{-j24t}+e^{-j16t}+e^{-j12t}+e^{-j8t}+e^{-j4t}+1+e^{j4t}+e^{j8t}+e^{j12t}+e^{j16t}+e^{j24t}+e^{j28t}+e^{j32t}$$

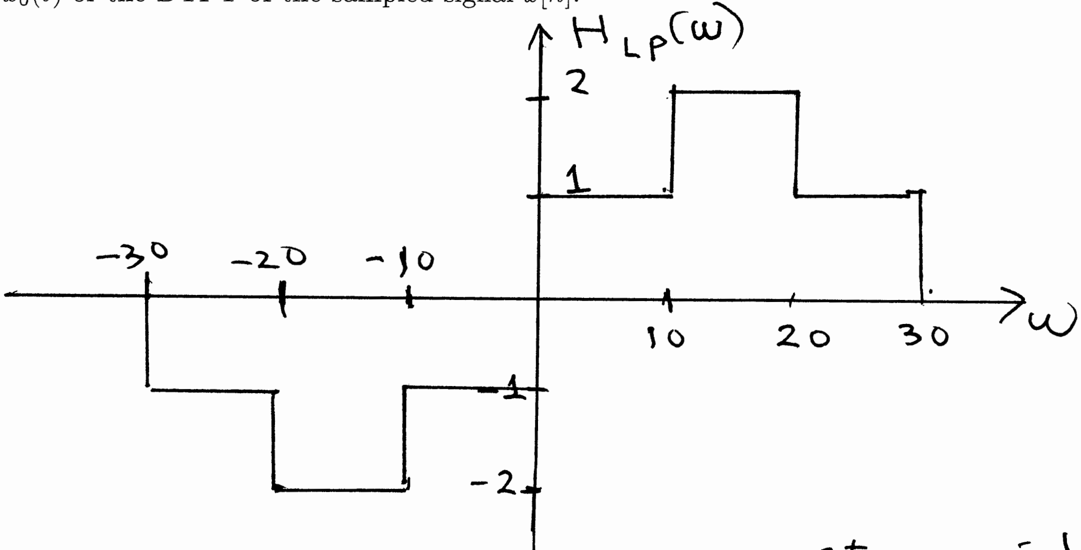
This signal is first input to an analog filter with impulse response

$$h_{LP}(t) = \left\{ \frac{\sin(15t)}{\pi t} + \frac{\sin(5t)}{\pi t} \right\} 2j\sin(15t)$$

to form $x(t) = x_0(t) * h_{LP}(t)$, and then $x(t)$ is sampled at a rate of $\omega_s = 64$ to form $x[n]$, so that the time between samples is $T_s = \frac{2\pi}{64}$. The DT signal $x[n]$ thus obtained is then input to a DT LTI system with impulse response

$$h[n] = 16 \left\{ \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \right\}^2 \qquad (1)$$

Show all work. Write your expression for the output $y[n] = x[n] * h[n]$ in the space below. Plot both the Fourier Transform of $h_{LP}(t)$ and the DTFT of $h[n]$ to help solve the problem, but since the input is a sum of sinewaves, it is not necessary to plot the Fourier Transform of $x_0(t)$ or the DTFT of the sampled signal $x[n]$.



$$x(t) = 0.1 + e^{j4t} + e^{j8t} + 2e^{j12t} + 2e^{j16t} + e^{j24t} + e^{j28t} + -e^{-4t} - e^{-j8t} - 2e^{-j12t} - 2e^{-j16t} - e^{-j24t} - e^{-j28t}$$

output of lowpass filter

$|k| \le 7$

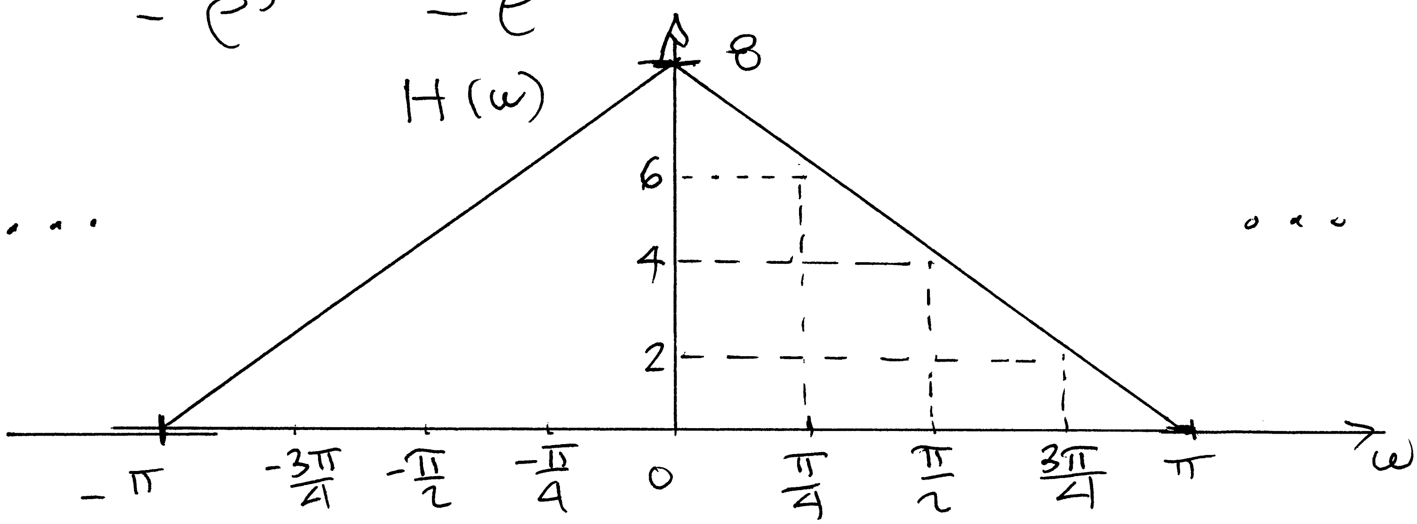
$k \neq 0$

$k \neq 5$

$$\omega_d = \omega_a T_s \Rightarrow k4\left(\frac{2\pi}{64}\right) = k\frac{\pi}{8}$$

Problem 2. You can continue your work for 2 here.

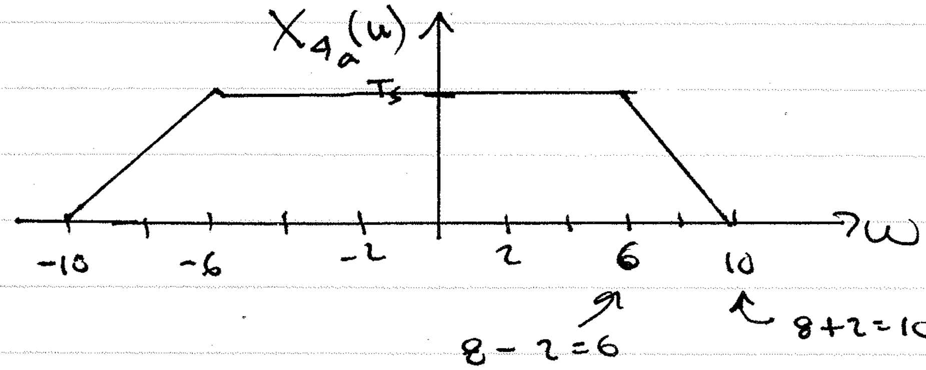
$$x[n] = e^{j\frac{\pi}{8}n} + e^{j\frac{2\pi}{8}n} + 2e^{j\frac{3\pi}{8}n} + 2e^{j\frac{4\pi}{8}n}$$
$$+ e^{j\frac{6\pi}{8}n} + e^{j\frac{7\pi}{8}n}$$
$$- e^{-j\frac{\pi}{8}} - e^{-j\frac{\pi}{4}n} - 2e^{-j\frac{3\pi}{8}n} - 2e^{-j\frac{\pi}{2}n}$$
$$- e^{-j\frac{3\pi}{4}n} - e^{-j\frac{7\pi}{8}n}$$



$$H\left(\frac{\pi}{8}\right) = 7 \quad H\left(\frac{3\pi}{8}\right) = 5 \quad H\left(\frac{7\pi}{8}\right) = 1$$

$$y[n] = 7e^{j\frac{\pi}{8}n} + 6e^{j\frac{2\pi}{8}n} + 10e^{j\frac{3\pi}{8}n} + 8e^{j\frac{4\pi}{8}n}$$
$$+ 2e^{j\frac{6\pi}{8}n} + e^{j\frac{7\pi}{8}n}$$
$$- 7e^{-j\frac{\pi}{8}n} - 6e^{-j\frac{2\pi}{8}n} - 10e^{-j\frac{3\pi}{8}n} - 8e^{-j\frac{4\pi}{8}n}$$
$$- 2e^{-j\frac{6\pi}{8}n} - e^{-j\frac{7\pi}{8}n}$$

$$x(t) = T_s \frac{\pi}{2} \left\{ \frac{\sin(2t)}{\pi t} \frac{\sin(8t)}{\pi t} \right\}$$

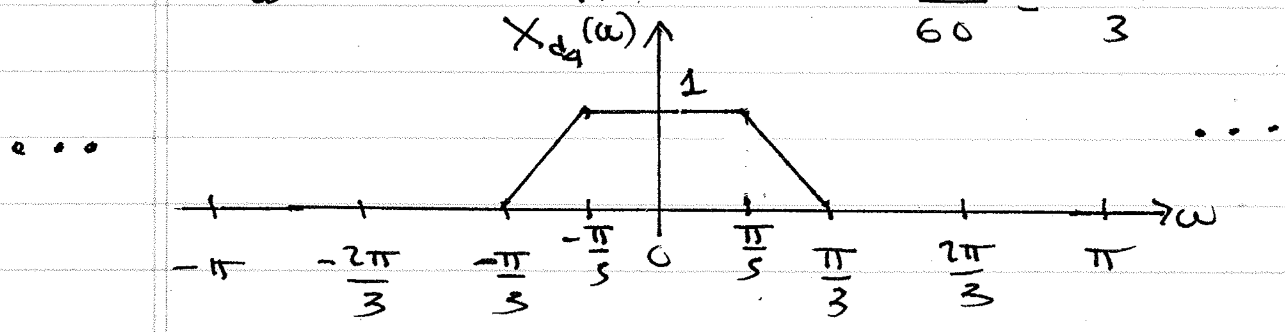


$\omega_{max} = 10$

$T_s = \frac{2\pi}{60}$ $\quad 60 > 2(\omega_{max}) = 20 \quad \Rightarrow$ no aliasing

$\omega_a = 6$ is mapped to $6 \frac{2\pi}{60} = \frac{\pi}{5}$

$\omega_a = 10$ is mapped to $10 \frac{2\pi}{60} = \frac{\pi}{3}$



(h) $T_s = \frac{2\pi}{18}$ $\quad 18 < 20 \Rightarrow$ aliasing starts at

$$(\omega_s - \omega_{max})T_s = (18-10)\frac{2\pi}{18}$$

See plots on next page

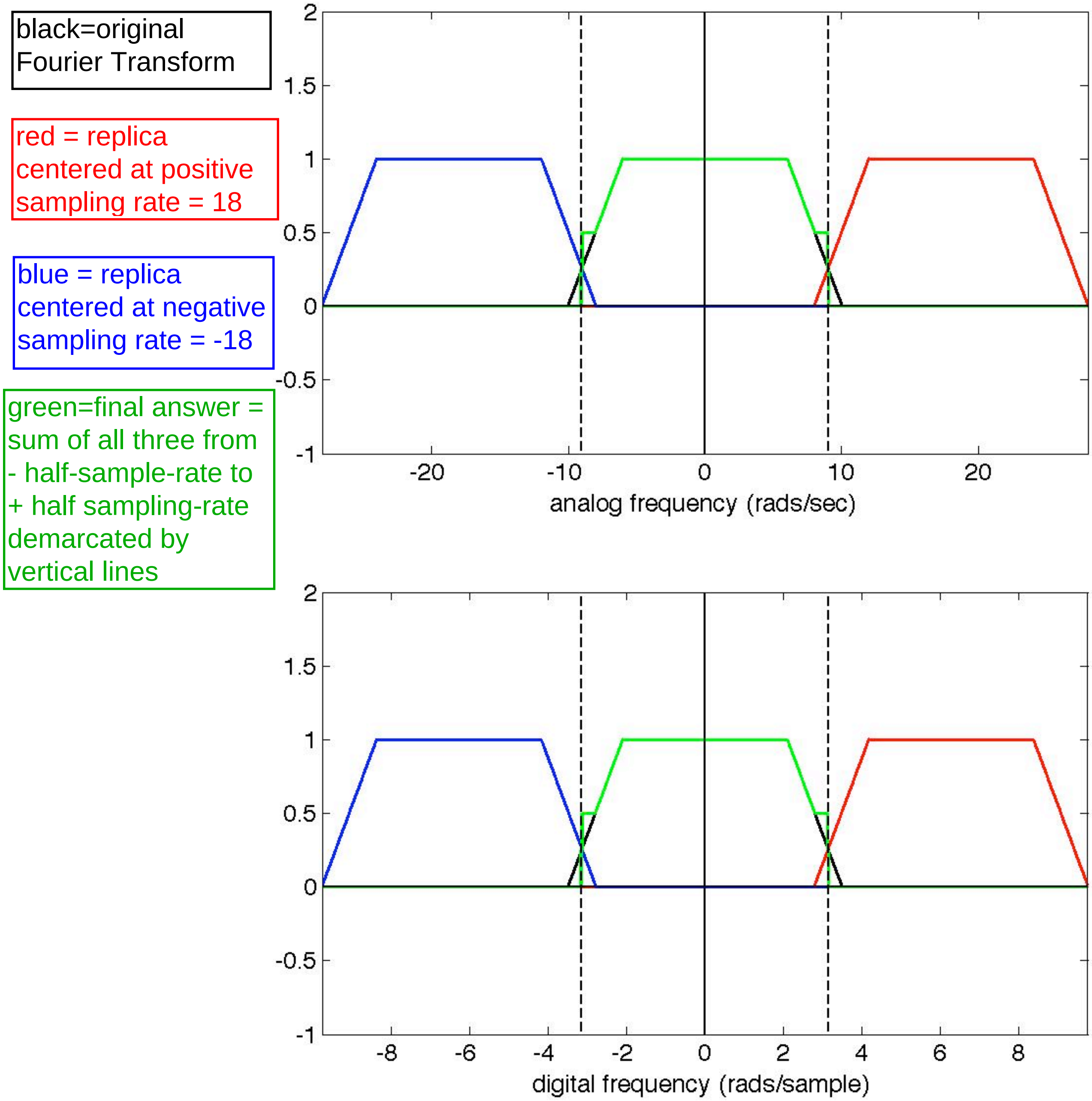
$$= \frac{8\pi}{9}$$

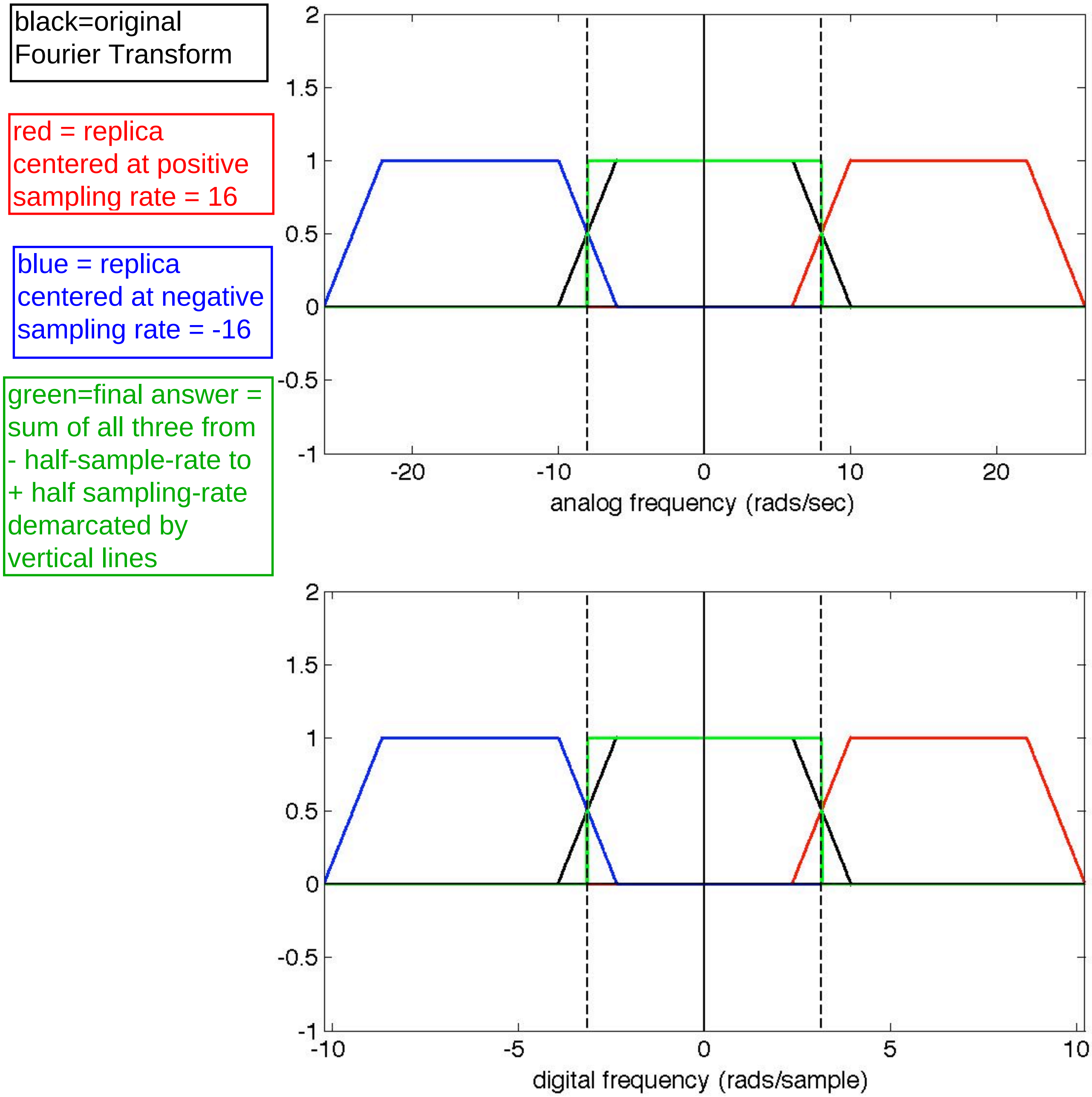
(i) $T_s = \frac{2\pi}{16}$ $\quad 16 < 20 \Rightarrow$ aliasing starts at

$$(\omega_s - \omega_{max})T_s = (16-10)\frac{2\pi}{16}$$

See plots on next page after next page

$$= \frac{3\pi}{4}$$


black=original Fourier Transform
red = replica centered at positive sampling rate = 18
blue = replica centered at negative sampling rate = -18
green=final answer = sum of all three from - half-sample-rate to + half sampling-rate demarcated by vertical lines
2
1.5
1
0.5
0
-0.5
-1
-20
-10
0
10
20
analog frequency (rads/sec)
2
1.5
1
0.5
0
-0.5
-1
-8
-6
-4
-2
0
2
4
6
8
digital frequency (rads/sample)


black=original Fourier Transform
red = replica centered at positive sampling rate = 16
blue = replica centered at negative sampling rate = -16
green=final answer = sum of all three from - half-sample-rate to + half sampling-rate demarcated by vertical lines
2
1.5
1
0.5
0
-0.5
-1
-20
-10
0
10
20
analog frequency (rads/sec)
2
1.5
1
0.5
0
-0.5
-1
-10
-5
0
5
10
digital frequency (rads/sample)