POINTS BREAKDOWN SOLUTION

1. 15 (3×5) 2. 25 $\begin{pmatrix} a. 15 \\ b. 10 \end{pmatrix}$ 3. 30 4. 30

NAME: **NAME**

EE301 Signals and Systems **In-Class Exam**

Exam 3 **Tuesday, Apr. 19, 2016**

Cover Sheet

Test Duration: 75 minutes.
Coverage: Chaps. 5,7
Open Book but Closed Notes.
One 8.5 in. x 11 in. crib sheet
Calculators NOT allowed.
All work should be done on the sheets provided.
You must show all work for each problem to receive full credit.
For Problem 4, plot your answers on the graphs provided.

VIP Note Regarding DTFT Plots: *The abscissa in each plot is the frequency axis. For each plot, the abscissa goes from -2π to 2π with tic marks every $\pi/8$. There is a dashed vertical line at $\omega = -\pi$ and another dashed vertical line at $\omega = +\pi$. You only have to plot any DTFT over $-\pi < \omega < \pi$.*

Problem 1. Short answer questions.

(a) Briefly write and explain one of the main advantages of digital over analog, in terms of storage, transmission, and/or processing, in a coherent sentence.

Ultimately, samples at equi-spaced instant in time are rounded-off and stored in terms of 0's and 1's This allows for the use of Error Control Coding through which we append additional 0's and 1's in such a way that we can automaticlly correct for errors in "reading" the bits off a CD/DVD

(b) If you sample at a rate ω_s in terms of radians/sec, what discrete-time frequency is the analog frequency $\frac{\omega_s}{2}$ mapped to? Then, briefly explain why π is the highest discrete-time frequency.

$$\omega_d = \omega_a T_s \qquad T_s = \frac{2\pi}{\omega_s}$$

$$= \omega_a \frac{2\pi}{\omega_s}$$

for $\omega_a = \frac{\omega_s}{2}$

$$= \frac{\omega_s}{2} \frac{2\pi}{\omega_s}$$

$$= \pi$$

Since $\frac{\omega_s}{2}$ is the highest frequency we can "see" or sense when we sample at a rate ω_s AND since $\frac{\omega_s}{2}$ is mapped to π => that's why π is the highest DT frequency

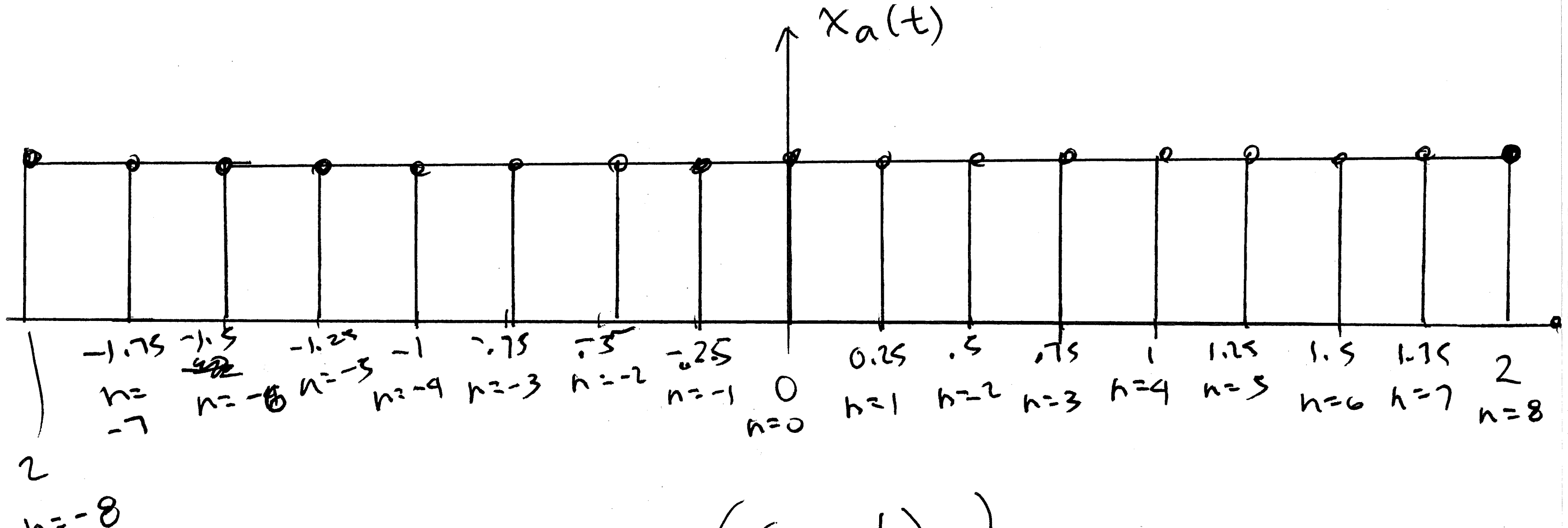
(c) The DT signal $x[n]$ is obtained by sampling the sinewave $x_{a_1}(t) = \cos(15t + \phi)$, where the value of the phase is $\phi = \pi/\sqrt{2}$, at a rate of $\omega_s = 40$ radians/sec. Specify the frequency ω_{a_2} of another analog sinewave $x_{a_2}(t) = \cos(\omega_{a_2} t + \phi)$ at a higher frequency that will yield the exact same DT signal $x[n]$ when sampled at the same rate, $\omega_s = 40$ radians/sec.

as discussed in class $15 + \ell 40$

where ℓ = integer

Problem 2 (a). The signal $x_a(t) = \{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.25$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a quarter of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), that is, at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.25 \text{ secs} \quad \text{and} \quad x_a(t) = \{u(t+2) - u(t-2)\}$$



$$X(\omega) = \frac{\sin\left(\left(8+\frac{1}{2}\right)\omega\right)}{\sin\left(\frac{1}{2}\omega\right)}$$

Table 5.2

Problem 2 (b). The signal $x_a(t) = t\{u(t+2) - u(t-2)\}$ is sampled every $T_s = 0.25$ seconds to form $x[n] = x_a(nT_s)$, where, again, T_s is a quarter of a second. Determine a closed-form expression for the DTFT $X(\omega)$ of the $x[n]$ thus obtained. Assume that the signal is turned on equal to 1 at both ends (edges), that is, at both $t = -2$ secs and $t = +2$ secs.

$$x[n] = x_a(nT_s) \quad \text{where: } T_s = 0.25 \text{ secs} \quad \text{and} \quad x_a(t) = t\{u(t+2) - u(t-2)\}$$

$$x[n] = nT_s \left\{ \underbrace{x_{2a}[n]}_{\text{from Prob. 2(a)}} \right\}$$

$$X(\omega) = \underbrace{T_s}_{=.25}\, j \frac{d}{d\omega}\left\{ \frac{\sin((8.5)\omega)}{\sin(.5\omega)} \right\}$$

Problem 3. Consider the input signal $x_p(t)$ below.

$$x_0(t) = e^{-j35t} + e^{-j25t} + e^{-j15t} + e^{-j5t} + 1 + e^{j5t} + e^{j15t} + e^{j25t} + e^{j35t}$$

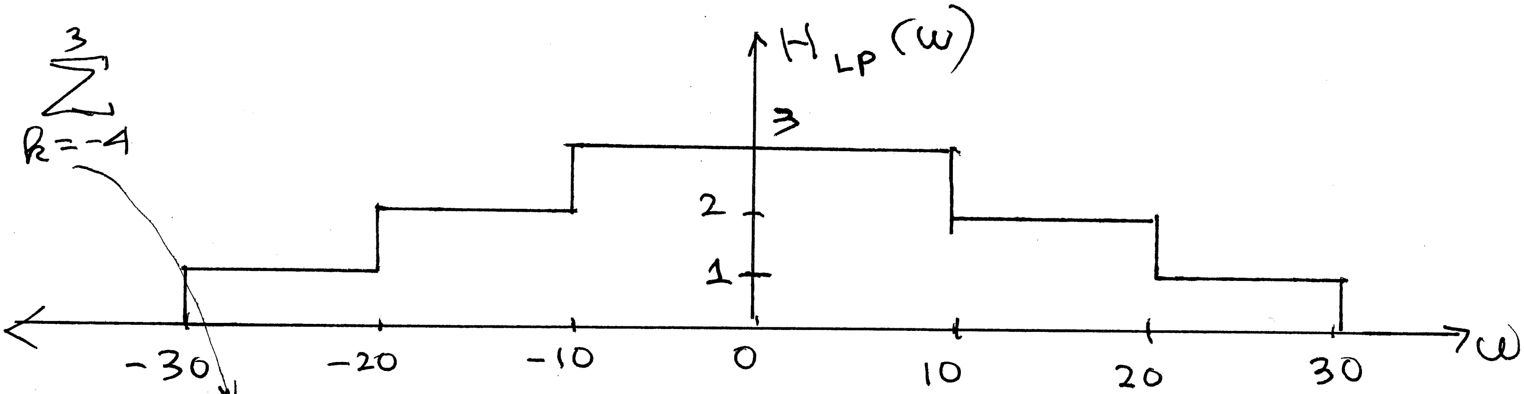
This signal is first input to an analog filter with impulse response

$$h_{LP}(t) = \frac{\sin(10t)}{\pi t} + \frac{\sin(20t)}{\pi t} + \frac{\sin(30t)}{\pi t}$$

to form $x(t) = x_0(t) * h_{LP}(t)$, and then $x(t)$ is sampled at a rate of $\omega_s = 50$ to form $x[n]$, so that the time between samples is $T_s = \frac{2\pi}{50}$. The DT signal $x[n]$ thus obtained is then input to a DT LTI system with impulse response

$$h[n] = \frac{\sin\left(\frac{\pi}{3}n\right)}{\pi n} + \frac{\sin\left(\frac{2\pi}{3}n\right)}{\pi n} \tag{1}$$

Show all work. Write your expression for the output $y[n] = x[n] * h[n]$ in the space below. Plot both the Fourier Transform of $h_{LP}(t)$ and the DTFT of $h[n]$ to help solve the problem, but since the input is a sum of sinewaves, it is not necessary to plot the Fourier Transform of $x_0(t)$ or the DTFT of the sampled signal $x[n]$.



$$x(t) = \sum_{k=-4}^{3} H_{LP}(k10+5)\, e^{j(k10+5)t} + H(0)\, 1$$

$$= 3 + 3\left(e^{j5t} + e^{-j5t}\right) + 2\left(e^{j15t} + e^{-j15t}\right)$$

$$+ e^{j25t} + e^{-j25t} \quad (\omega = 35 \text{ is zeroed out})$$

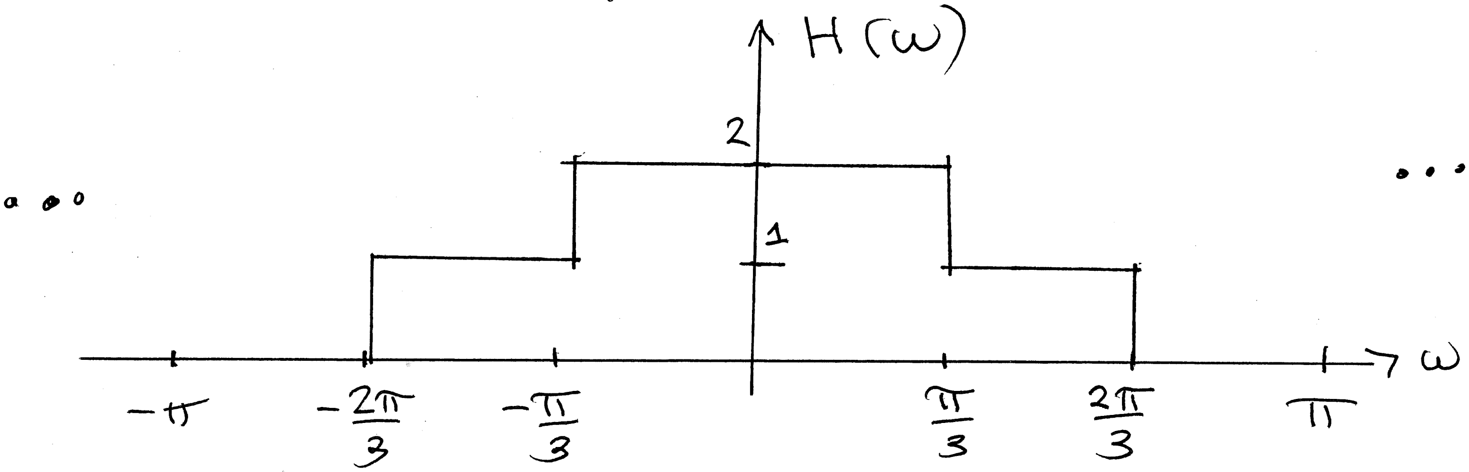
Time between samples $T_s = \frac{2\pi}{50} = \frac{\pi}{25}$

Digital frequencies: $\omega_d = \omega_a T_s = (5 + k10)\frac{\pi}{25}$ $\qquad + \omega = 0$

$$= \frac{\pi}{5} + k\frac{2\pi}{5}$$

$$-3 \le k \le 2$$

Problem 3. You can continue your work for 3 here.



$$x[n] = 3 + 3\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right) + 2\left(e^{j\frac{3\pi}{5}n} + e^{-j\frac{3\pi}{5}n}\right) + e^{j\pi n} + e^{-j\pi n}$$

$$y(n) = (2)3 + (2)3\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right) + (1)2\left(e^{j\frac{3\pi}{5}n} + e^{-j\frac{3\pi}{5}n}\right) + (0)\left(e^{j\pi n} + e^{-j\pi n}\right)$$

$$= 6 + 6\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right) + 2\left(e^{j\frac{3\pi}{5}n} + e^{-j\frac{3\pi}{5}n}\right)$$

Workout Problem 4. Consider the continuous-time signal $x_a(t)$ below. Note that the the Fourier Transform $X_a(\omega)$ is purely real-valued, and the multiplication by the scalar T_s is intended to offset the amplitude-scaling by the sampling rate $F_s = \frac{1}{T_s}$ that inherently occurs in the process of sampling.

$$x_a(t) = T_s \left\{ \frac{\sin(10t)}{\pi t} + \frac{\sin(20t)}{\pi t} + \frac{\sin(30t)}{\pi t} \right\}$$

(a) A discrete-time signal is created by sampling $x_a(t)$ according to $x[n] = x_a(nT_s)$ for $T_s = \frac{2\pi}{60}$. Plot the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work on this page and the next page, and do your plot in the space provided on the next page.

(b) Repeat part (a) for $T_s = \frac{2\pi}{50}$. Plot the new DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show your work and do your plot in the space provided on the sheets attached.

(a) $x_a(t) = h_{LP}(t)$ from Prob. 3

$\omega_M = 30$ Nyquist rate $= \omega_N = 2\omega_M$

$= 60$

Sampling right at Nyquist rate compresses $X_a(\omega) = \underbrace{H_{LP}(\omega)}_{\text{Prob. 3}}$ to exact fit in $-\pi < \omega < \pi$

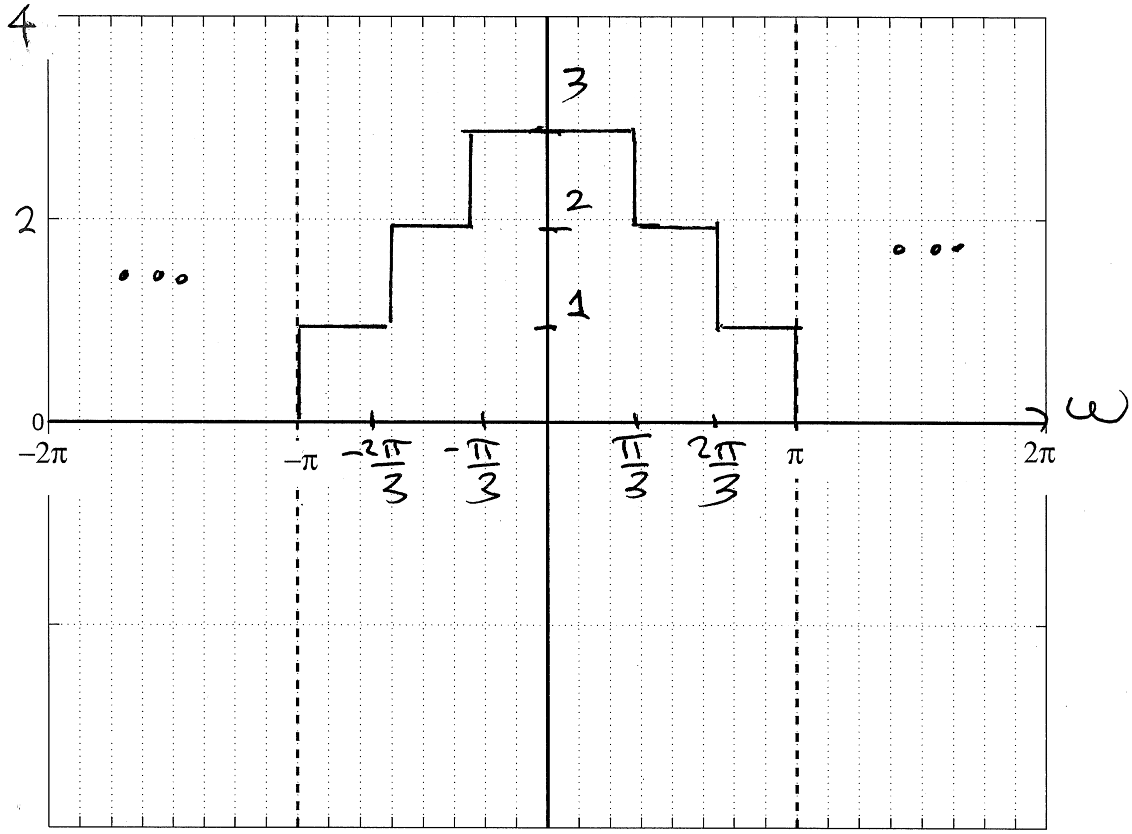
(b) $T_s = \frac{2\pi}{50}$ $\omega_s = 50 < 60$ $\Rightarrow$ Aliasing

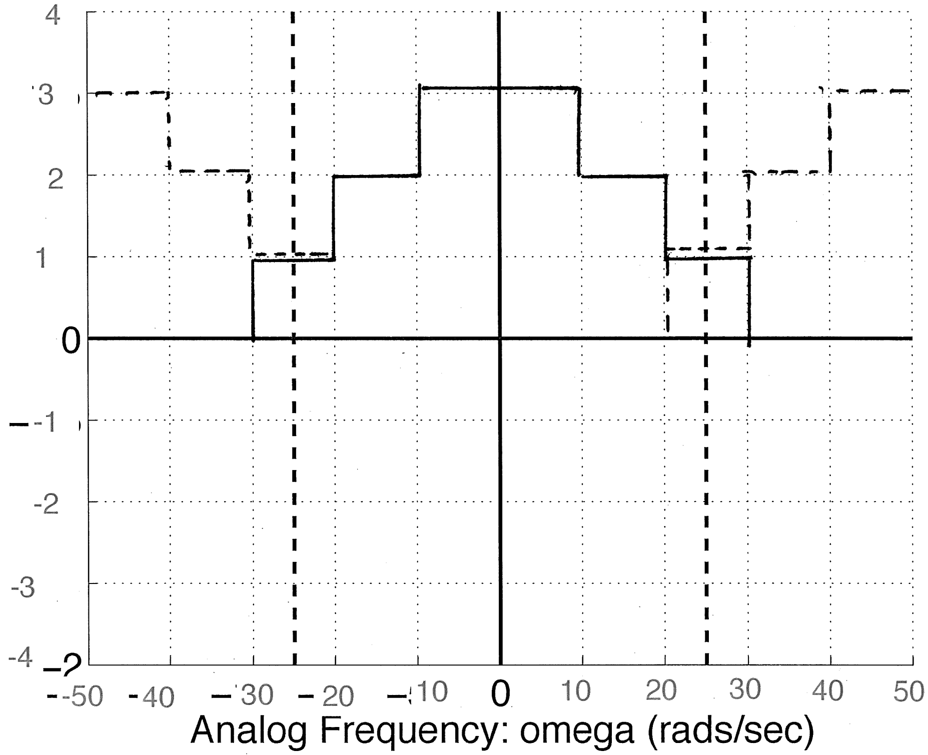
aliasing starts at $\omega_s - \omega_M = 50 - 30 = 20$

and this gets mapped to the digital frequency

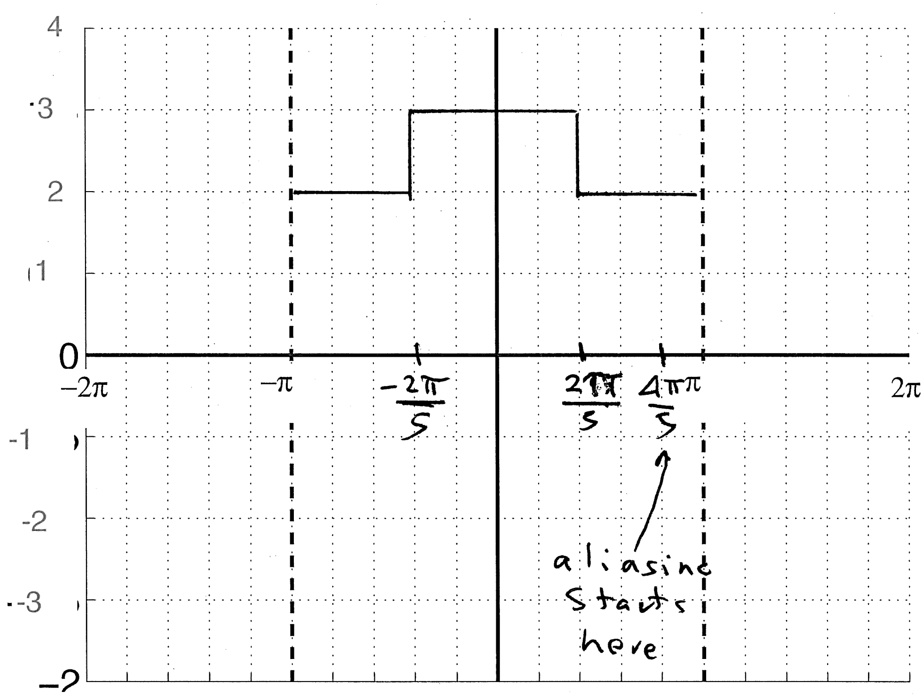
$20 \cdot \frac{2\pi}{50} = \frac{4\pi}{5}$

Plot your answer to Problem 4 (a) here. Show work .
Plot your answer to Problem 4 (b) on next page .

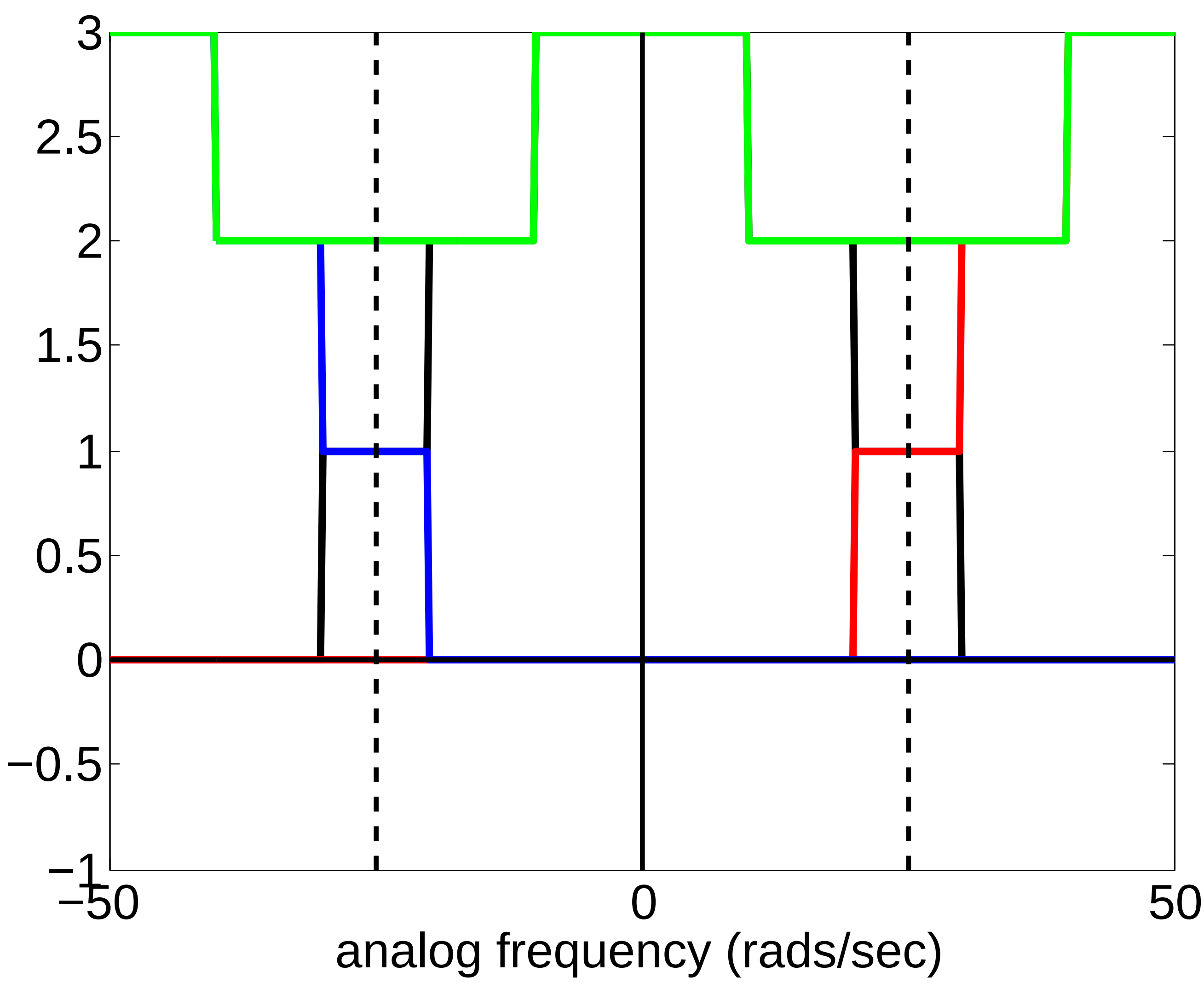




Analog Frequency: omega (rads/sec)



−2π
−π
−2π/5
2π/5
4π/5
π
2π
aliasing starts here
Digital Frequency: omega (rads/sec)



3
2.5
2
1.5
1
0.5
0
−0.5
−1
−50
0
50
analog frequency (rads/sec)